How many distractors are needed in a multiple choice question?

Background

A course did a pilot study using multiple-choice questions in one-third of the exam. It was successful and they now want to mainstream this and redraft the specimen, as well as writing the next two papers.

They asked the following questions:

- If we want 3 correct answers, what's the recommended number of distractors?
- If we want 4 correct answers, what's the recommended number of distractors?
- If we want 5 correct answers, what's the recommended number of distractors?

The type of question they intend using is one that allows them to assess the content of an entire block. No credit will be given for partially correct answers and the responses are designed to be either correct or incorrect.

Method

The following mathematical formula gives the number of ways in which you can choose *r* items from a list of *n* options.

$$\frac{n!}{(n-r)!\times r!}$$

It follows from this that if there are *c* correct answers and *d* distractors, then the number of ways of choosing *c* answers from *c*+*d* options is given by:

$$\frac{(c+d)!}{c!\times d!}$$

And because there is just one correct combination, the probability of choosing that combination is one over the number of ways of choosing *c* answers from *c*+*d* options.

$$p = \frac{c!\times d!}{(c+d)!}$$

The table below uses this formula to compute the probability of a student getting such a question right purely by random guessing.

Number of distractors	Three correct options	Four correct options	Five correct options
1	0.250	0.200	0.167
2	0.100	0.067	0.048
3	0.050	0.029	0.018
4	0.028	0.014	0.008
5	0.018	0.008	0.004
6	0.012	0.005	0.002

The effect of multiple questions

It is clear that the random guess probability should be considerably less than any "pass mark", but defining an absolute value is not quite so easy.

If the student has to answer n questions, each of which has a probability p of being guessed correctly, then if they get one mark for each correct question their mean score will be

$$n \times p$$

with a standard deviation of

$$\sqrt{n \times p \times (1 - p)}$$

An example may make this clearer. If there are ten questions each of which has three correct options and two distractors, then the table above shows that each question has a random guess probability of 0.100.

The mean score expected by random guessing ten questions is then

$$10 \times 0.100 = 1.00$$

with a standard deviation of

$$\sqrt{10 \times 0.1 \times 0.9} = 0.95$$

In this case we have a binomial distribution, and when numbers are large, a binomial can be approximated by the normal distribution. In this case the numbers are not really large enough, but to get a feel for things, let's assume that such an approximation is valid.

With a normal distribution, a little over 2% of values will be more than than two standard deviations above the mean (2.90), while only about 0.1% of values would be expected to exceed three standard deviations above the mean (3.85). So if the "pass mark" for such a set of questions had been set at 40%, then you would be really quite confident that students would not be able to pass simply by guessing.

Similar checks can be used to see whether any particular number of questions with a given random guess probability would be acceptable.

For more information, see http://www.open.ac.uk/blogs/SallyJordan/?p=482