Mathematics, Computing and Technology/Science

MST224
DIAGNOSTIC QUIZ

Am I ready to start on MST224?

The diagnostic quiz below is designed to help you to answer this question. This document also contains some advice on preparatory work that you may find useful before starting on MST224 (see below and pages 9–10).

The mathematical skills required for MST224 can be separated into two levels:

A those that are assumed but not discussed at all in the module;

B those that are reviewed in the *Unit 1* of the module.

The diagnostic quiz below is divided into corresponding sections A and B.

To be ready to start on MST224, you should be confident about level A topics. You should also have met level B topics before, and be able to handle them with the brief reminder provided in *Unit 1* of the module.

Try the questions now, and then see the notes on pages 9–10 of this booklet to see if you are ready for MST224.

Do contact your Student Support Team via StudentHome if you have any queries about your suitability for the module.

Diagnostic Quiz – Questions

LEVEL A

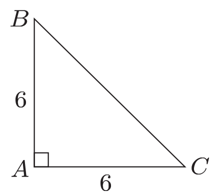
1 Using a calculator, give the values of each of the following to three decimal places:

(a) $\tan(1.2)$ (where 1.2 is in radians);

(b) $e^{-2.731}$;

(c) $\ln(4/27)$.

2 In the triangle BAC, the angle BAC is a right angle, and the sides AB and AC are each of length 6.

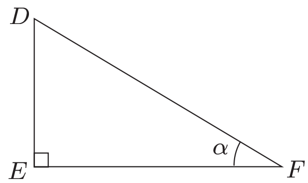


(a) Give each of the angles in triangle ABC in degrees and in radians.

(b) What is the length BC?

(c) What is the area of triangle ABC?

3 (a) In the triangle DEF, the angle DEF is a right angle, and angle EFD is α. Write down each of $\cos\alpha$, $\sin\alpha$ and $\tan\alpha$ as ratios of sides in the triangle DEF.



(b) Give the values of $\cos(180°)$ and $\sin(270°)$.

4 Solve for x each of the following equations.

(a) $3x + 4 = 10$

(b) $3(x+3) - 7(x-1) = 0$

(c) $\dfrac{2}{1+x} = \dfrac{3}{2-x}$

(d) $\sqrt{x^2+7} = 4$

5 (a) Make t the subject of the equation

$$x = x_0 - \tfrac{1}{2}gt^2.$$

(b) Make x the subject of the equation

$$\sqrt{\frac{x-2}{x+3}} = t.$$

6 Give the equation of the straight line passing through the points $y = 2$ when $x = 0$ and $y = 8$ when $x = 2$. What is the gradient of this line?

7 If $y(x) = 3 + 2x - \sin(2x)$, what is $y\left(\frac{\pi}{2}\right)$?

8 (a) Solve for y the equation

$$2y^2 - 4y + 1 = 0.$$

(b) Solve for λ the equation

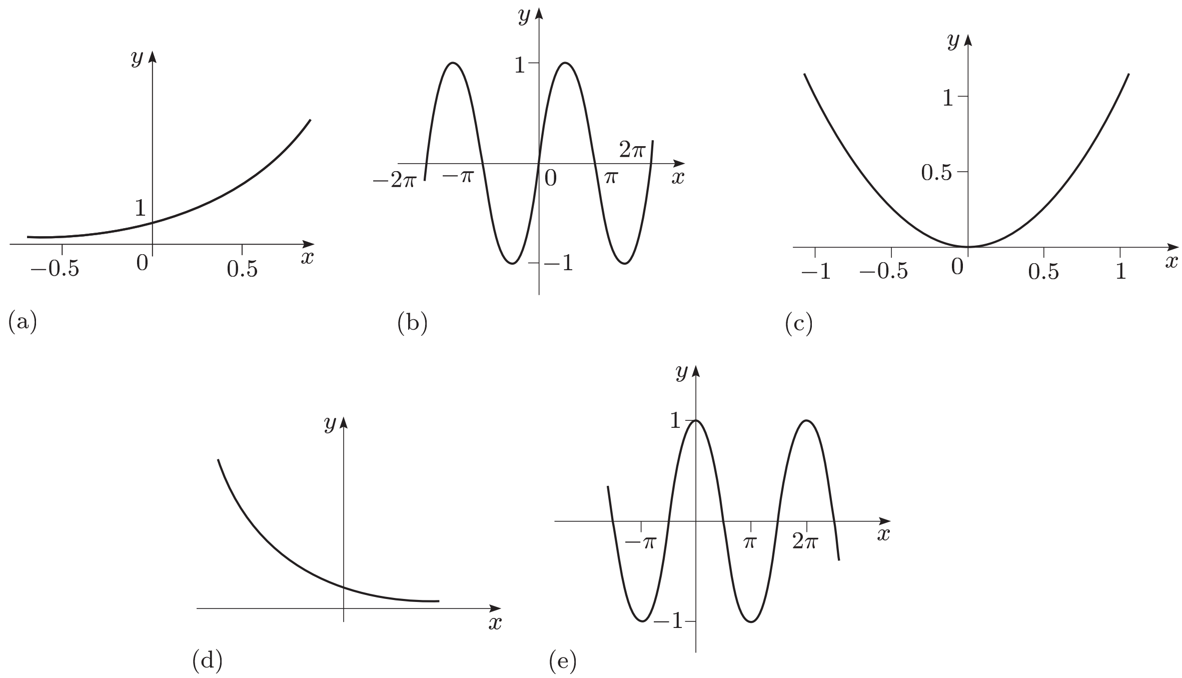
$$\lambda^2 + 4\lambda + 4 = 0.$$

9 Solve the following simultaneous equations for x and y:

$$2x - y = 3,$$
$$3x + y = 2.$$

10 Five graphs are given in parts (a)–(e) of the figure below. Each graph is that of one of the functions (i)–(v). Match each graph with the appropriate function.

Functions: (i) $y(x) = e^{-x}$, (ii) $y(x) = e^{2x}$, (iii) $y(x) = \sin x$, (iv) $y(x) = \cos x$, (v) $y(x) = x^2$.



11 Simplify each of the following.

(a) x^2x^5 (b) x^3/x^4 (c) $(x^2)^3$ (d) $9^{1/2}$

12 If $|x - 2| < 10^{-2}$, what range of values can x take?

13 Solve for ν the equation below (where $m \neq 0$):

$$-\frac{m}{gr} = -\frac{\mu m}{\nu^2}.$$

LEVEL B

14 Express $(e^{-2x} \times e^{3x})^2$ in the form e^y.

15 Express $\frac{1}{2}\ln(25) + 3\ln(\frac{1}{2})$ in the form $\ln(y)$.

16 Show that $y = \ln(2e^{-x/2})$ is the equation of a straight line. What is the gradient of this line?

17 Solve for y the equation

$$\ln(y) = 2\ln(x) - 1.$$

18 (a) What solutions for x has the equation $\sin x = 1$?

(b) What value does your calculator give for $\arcsin(1)$?

19 What is $\cos^2\alpha + \sin^2\alpha$ (where α may be any real number)?

20 Use the trigonometric identity $\cos(a+b) = \cos a \cos b - \sin a \sin b$, and particular values of a and b, to simplify $\cos(\pi + x)$.

21 Find the local maxima and minima of the function

$$y(x) = 2x^3 - 3x^2 - 12x + 6.$$

22 (a) Find $\dfrac{ds}{dt}$ where $s = 5e^{3t}$.

(b) Find $y'(t)$ where $y(t) = 3t^5 - 10\sqrt{t}$.

(c) Find $\dfrac{dz}{dx}$ where $z = 14\sin(x/8)$.

23 Evaluate each of the following integrals.

(a) $\displaystyle\int (1 + 6x^3)\,dx$ (b) $\displaystyle\int_0^{\pi} \sin(3t)\,dt$

24 Suppose that

$$-\frac{m}{gr} \geq -\frac{\mu m}{\nu^2},$$

where m, r, g and μ are positive.

(a) Rearrange this inequality by multiplying each side first by $-\nu^2$, then by gr/m.

(b) In terms of the other parameters, what is the largest value that ν can take?

25 (a) (i) Find $y'(t)$ where $y(t) = t\sin(3t)$.

(ii) Find $\dfrac{dx}{dt}$ where $x = \ln(t^3 + 1)$.

(b) Find the velocity at time $t = 3$ of an object whose position at time t is given by $x(t) = e^{-2t}\cos(\frac{\pi}{3}t)$.

26 (a) Use integration by substitution to find $\int x^2 \exp(2+3x^3)\,dx$.

(b) Use integration by parts to find $\int x \ln x \, dx$.

27 Express the complex number $(1+i)(3+2i)$ in the form $a+bi$.

28 Find the imaginary part of the complex number $(e^{2+i\pi})^3$.

Diagnostic Quiz – Answers

LEVEL A

1 (a) 2.572

(b) 0.065

(c) −1.910

2 (a) $\angle ABC = \angle ACB = 45^\circ = \frac{\pi}{4}$ radians.

$\angle BAC = 90^\circ = \frac{\pi}{2}$ radians.

(b) By Pythagoras's Theorem,

$$BC^2 = AB^2 + AC^2 = 6^2 + 6^2 = 72,$$

so

$$BC = \sqrt{72} = 6\sqrt{2}.$$

(c) The area of a triangle equals half its base times its height, so the area of triangle ABC is

$$\tfrac{1}{2} \times 6 \times 6 = 18 \text{ square units.}$$

3 (a) $\cos\alpha = \dfrac{EF}{DF}$, $\sin\alpha = \dfrac{DE}{DF}$, $\tan\alpha = \dfrac{DE}{EF}$.

(b) $\cos(180^\circ) = -1$, $\sin(270^\circ) = -1$.

4 (a) $3x + 4 = 10$

$3x = 6$

$x = 2$

(b) $3(x+3) - 7(x-1) = 0$

$3x + 9 - 7x + 7 = 0$

$-4x + 16 = 0$

$x = 4$

(c) $\dfrac{2}{1+x} = \dfrac{3}{2-x}$

$2(2-x) = 3(1+x)$

$4 - 2x = 3 + 3x$

$5x = 1$

$x = \frac{1}{5}$

(d) $\sqrt{x^2+7} = 4$

$x^2 + 7 = 4^2 = 16$

$x^2 = 9$

$x = \pm\sqrt{9}$

So $x = 3$ or $x = -3$.

5 (a) $x = x_0 - \frac{1}{2}gt^2$

$gt^2 = 2(x_0 - x)$

$t = \pm\sqrt{\dfrac{2}{g}(x_0 - x)}$

(b) $\sqrt{\dfrac{x-2}{x+3}} = t$

$\dfrac{x-2}{x+3} = t^2$

$x - 2 = t^2(x+3) = t^2x + 3t^2$

$x(1-t^2) = 2 + 3t^2$

$x = \dfrac{2+3t^2}{1-t^2}$

6 The equation of a straight line has the form

$$y = mx + c.$$

To satisfy the given conditions, the constants m and c must satisfy the equations

$2 = c$ (since $y = 2$ when $x = 0$),

$8 = 2m + c$ (since $y = 8$ when $x = 2$).

Thus $c = 2$ and $m = 3$, so the required equation is

$$y = 3x + 2.$$

The gradient of this line is given by m, and so is 3.

7
$$\begin{aligned} y\left(\tfrac{\pi}{2}\right) &= 3 + 2 \times \tfrac{\pi}{2} - \sin\left(2 \times \tfrac{\pi}{2}\right) \\ &= 3 + \pi - \sin\pi \\ &= 3 + \pi - 0 \\ &= 3 + \pi \end{aligned}$$

8 Use the formula for solving a quadratic equation.

(a)
$$\begin{aligned} y &= \frac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} \\ &= \frac{4 \pm \sqrt{8}}{4} = 1 \pm \tfrac{1}{2}\sqrt{2} \end{aligned}$$

(b) $\lambda = \dfrac{-4 \pm \sqrt{16-16}}{2} = -2$

(The expression $\lambda^2 + 4\lambda + 4$ is a perfect square, $(\lambda+2)^2$.)

9 Adding the equations gives $5x = 5$, so $x = 1$. Then the first equation gives $2 - y = 3$, so $y = -1$.

The solution is $x = 1$, $y = -1$.

10 The matching is as follows.

(a)(ii) (b)(iii) (c)(v) (d)(i) (e)(iv)

11 We use the rules for manipulating indices.

(a) $x^2 x^5 = x^{2+5} = x^7$

(b) $x^3/x^4 = x^{3-4} = x^{-1} (= 1/x)$

(c) $(x^2)^3 = x^{2\times 3} = x^6$

(d) $9^{1/2} = \sqrt{9} = 3$

12 Recall that $|y|$ means y if $y \geq 0$, and $-y$ if $y < 0$. If $|x-2| < 10^{-2}$, then $-10^{-2} < x - 2 < 10^{-2}$, so $2 - 10^{-2} < x < 2 + 10^{-2}$, i.e. $1.99 < x < 2.01$.

13 If $-\dfrac{m}{gr} = -\dfrac{\mu m}{\nu^2}$, then multiplying each side by $-\dfrac{\nu^2}{m}$ gives

$$\left(-\frac{\nu^2}{m}\right)\left(-\frac{m}{gr}\right) = \left(-\frac{\nu^2}{m}\right)\left(-\frac{\mu m}{\nu^2}\right),$$

i.e. $\dfrac{\nu^2}{gr} = \mu$. Hence $\nu = \pm\sqrt{\mu g r}$.

LEVEL B

14 $(e^{-2x} \times e^{3x})^2 = (e^{3x-2x})^2 = (e^x)^2 = e^{2x}$

[*Unit 1*, Subsection 1.4]

15
$$\begin{aligned}\tfrac{1}{2}\ln(25) + 3\ln(\tfrac{1}{2}) &= \ln(\sqrt{25}\,) + \ln((\tfrac{1}{2})^3) \\ &= \ln(5) + \ln(\tfrac{1}{8}) \\ &= \ln(\tfrac{5}{8})\end{aligned}$$

[See MST224 *Unit 1*, Subsection 2.2]

16 Using the properties of exponentials and logarithms,

$$\ln(2e^{-x/2}) = \ln(2) + \ln(e^{-x/2}) = \ln(2) - \tfrac{1}{2}x,$$

so

$$y = \ln(2) - \tfrac{1}{2}x.$$

This is the equation of a straight line.

The gradient of the straight line is $-\frac{1}{2}$ (the coefficient of x).

[See MST224 *Unit 1*, Subsections 1.1 and 2.2]

17 If $\ln(y) = 2\ln(x) - 1$, then taking exponentials of each side gives

$$\begin{aligned}\exp(\ln(y)) &= \exp(2\ln(x) - 1) \\ y &= \exp(\ln(x^2) - 1) \\ &= \exp(\ln(x^2))/\exp(1) \\ &= x^2/e.\end{aligned}$$

[See MST224 *Unit 1*, Subsection 2.2]

18 (a) $\sin x = 1$ when $x = \frac{\pi}{2}$, or when x differs from $\frac{\pi}{2}$ by a multiple of 2π.

(b) My calculator gives $\arcsin(1) = 90$, but that is because it is working in degrees. If your calculator is working in radians (as it will need to be for MST224), then it should give

$$\arcsin(1) = 1.570796327 \quad (\text{i.e. } \tfrac{\pi}{2}).$$

[See MST224 *Unit 1*, Subsections 3.1 and 3.2]

19 $\cos^2\alpha + \sin^2\alpha = 1$.

[See MST224 *Unit 1*, Subsection 3.3]

20 We have

$$\begin{aligned}\cos(\pi + x) &= \cos\pi\cos x - \sin\pi\sin x \\ &= (-1)\cos x - (0)\sin x \\ &= -\cos x.\end{aligned}$$

[See MST224 *Unit 1*, Subsection 3.3]

21 To find local maxima and minima, first find the stationary points, where $\dfrac{dy}{dx} = 0$.

Differentiating $y = 2x^3 - 3x^2 - 12x + 6$ gives

$$\frac{dy}{dx} = 6x^2 - 6x - 12.$$

So to find the stationary points, solve

$$6x^2 - 6x - 12 = 0,$$

i.e.

$$x^2 - x - 2 = 0.$$

To solve this quadratic equation, you can either use the formula, or factorize to obtain

$$(x-2)(x+1) = 0.$$

Thus there are stationary points at $x = 2$ and $x = -1$.

Now $\dfrac{d^2y}{dx^2} = 12x - 6$.

At $x = -1$, this is negative, so there is a local maximum at $x = -1$, of value $y = 13$.

At $x = 2$, this second derivative is positive, so there is a local minimum at $x = 2$, of value $y = -14$.

[See MST224 *Unit 1*, Subsection 5.3]

22 (a) If $s = 5e^{3t}$, then

$$\frac{ds}{dt} = 5(3e^{3t}) = 15e^{3t}.$$

(b) If $y(t) = 3t^5 - 10\sqrt{t}$, then

$$y'(t) = 15t^4 - 5t^{-1/2}.$$

(c) If $z = 14\sin(x/8)$, then

$$\frac{dz}{dx} = \tfrac{14}{8}\cos(x/8) = \tfrac{7}{4}\cos(x/8).$$

[See MST224 *Unit 1*, Subsection 5.2]

23 (a) This is an indefinite integral:

$$\int (1+6x^3)\,dx = x + \tfrac{6}{4}x^4 + c = x + \tfrac{3}{2}x^4 + c,$$

where c is an arbitrary constant.

(b) This is a definite integral:

$$\int_0^\pi \sin(3t)\,dt = \left[-\tfrac{1}{3}\cos(3t)\right]_0^\pi = -\tfrac{1}{3}(\cos(3\pi) - \cos(0)) = -\tfrac{1}{3}(-1-1) = \tfrac{2}{3}.$$

[See MST224 *Unit 1*, Subsection 6.2]

24 We have

$$-\frac{m}{gr} \geq -\frac{\mu m}{\nu^2}.$$

(a) The quantity $-\nu^2$ is negative, so on multiplying both sides by $-\nu^2$, we must reverse the inequality:

$$\frac{m}{gr}\nu^2 \leq \mu m.$$

Then (since gr/m is positive)

$$\nu^2 \leq \mu gr.$$

(b) The largest value that ν can take is $\sqrt{\mu gr}$.

[See MST224 *Unit 1*, Subsection 5.2]

25 (a) (i) To differentiate $y(t) = t\sin(3t)$, use the product rule. We obtain

$$y'(t) = \sin(3t) + 3t\cos(3t).$$

(ii) To differentiate $x = \ln(t^3+1)$, use the composite (or 'function of a function') rule. We obtain

$$\frac{dx}{dt} = 3t^2 \times \frac{1}{t^3+1} = \frac{3t^2}{t^3+1}.$$

(b) To find the velocity $v(t)$ of the object, we differentiate the expression for its position, i.e.

$$v(t) = \frac{dx}{dt} = -2e^{-2t}\cos(\tfrac{\pi}{3}t) - \tfrac{\pi}{3}e^{-2t}\sin(\tfrac{\pi}{3}t) = -e^{-2t}(2\cos(\tfrac{\pi}{3}t) + \tfrac{\pi}{3}\sin(\tfrac{\pi}{3}t)).$$

So at time $t = 3$, the object's velocity is

$$v(3) = -e^{-6}(2\cos\pi - \tfrac{\pi}{3}\sin\pi) = -e^{-6}(2(-1) + \tfrac{\pi}{3}(0)) = 2e^{-6} \simeq 0.005.$$

[See MST224 *Unit 1*, Subsection 5.2]

26 (a) Integration by substitution uses the formula

$$\int f(u)\frac{du}{dx}\,dx = \int f(u)\,du.$$

With $u(x) = 2 + 3x^3$, we have $\dfrac{du}{dx} = 9x^2$, and then

$$\begin{aligned}\int x^2 \exp(2+3x^3)\,dx &= \int \tfrac{1}{9}\frac{du}{dx}\exp u\,dx\\ &= \tfrac{1}{9}\int \exp u\,\frac{du}{dx}\,dx\\ &= \tfrac{1}{9}\int \exp u\,du\\ &= \tfrac{1}{9}\exp u + c\\ &= \tfrac{1}{9}\exp(2+3x^3) + c,\end{aligned}$$

where c is an arbitrary constant.

(b) Integration by parts uses the formula

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

With $f(x) = \ln x$ and $g(x) = \frac{1}{2}x^2$, we have $f'(x) = 1/x$ and $g'(x) = x$, and then

$$\int x\ln x\,dx = \tfrac{1}{2}x^2\ln x - \int \tfrac{1}{2}x\,dx = \tfrac{1}{2}x^2\ln x - \tfrac{1}{4}x^2 + c,$$

where c is an arbitrary constant.

[See MST224 *Unit 1*, Subsection 6.3]

27 Since $i^2 = -1$, we have

$$(1+i)(3+2i) = 3 + 2i + 3i + 2i^2 = 3 + 5i - 2 = 1 + 5i.$$

[See MST224 *Unit 1*, Subsection 4.1]

28 We have

$$(e^{2+i\pi})^3 = e^{(2+i\pi)\times 3} = e^{6+3i\pi} = e^6(\cos(3\pi) + i\sin(3\pi)).$$

The imaginary part of this is

$$e^6\sin(3\pi) = 0.$$

[See MST224 *Unit 1*, Subsection 4.2]

Materials that you can use to prepare for MST224

MST224 Bridging Material

This is designed for students who have done MST121 (or equivalent) instead of MST124. So long as you are familiar with the calculus in MST121 (or its equivalent), then this Bridging Material should take two or three weeks to study. This topics revised, in order of importance to MST224, are:

- techniques for differentiating products, quotients and composite functions;
- integration methods, namely integration by parts and by substitution;
- complex numbers.
- techniques for approximating functions using Taylor polynomials;

MST224 expects you to be proficient at differentiation 'by hand'. The integration methods are needed less often as most of the integrals can be done by using the table of standard integrals given in the course Handbook.

The MST224 Bridging Material is available on your qualification website and on the MST121 module website.

Unit 1 of MST224

This reviews a number of topics used in MST224. This unit does not introduce these topics from scratch; rather, it provides a reminder of them, and an opportunity to refresh your memory and practise techniques. Topics covered include:

- standard functions, such as linear, quadratic, exponential and logarithm functions, and algebraic manipulations involving these;
- trigonometric functions and identities involving these;
- complex numbers;
- differentiation;
- integration;

This unit includes the topics in the Bridging Material, but the Bridging Material covers them more thoroughly.

Later units of MST224 expect you to be proficient in the ideas and methods covered in *Unit 1*, particularly the use of the various standard functions, including the trigonometric functions, and manipulation of expressions involving them, differentiation, and integration using the table of standard integrals given in the course Handbook.

Unit 1 of MST224 is included in the sample materials on the qualification website.

What can I do to prepare for MST224?

Use of the following flowchart in connection with the diagnostic quiz should help you to decide whether MST224 is an appropriate course for you, and what you should do by way of preparation before the course starts.

Most or all of level A is completely unfamiliar.	Request an interview with a Regional Advisor. Consider MU123 or MST124.
Some of levels A and B is familiar, but some is not.	Consider MST124.
I have met nearly all the topics in levels A and B before, but need some practice.	You will need to spend some time in preparing for MST224. Study the MST224 Bridging Material. Also, start work on *Unit 1* as soon as possible, so that you can spend more than one week on it if need be.
I have met all the topics in the quiz before, but need to practise some of them.	Start work on *Unit 1* as soon as possible, so that you can spend more than one week on it if need be.
I think that I am already fully prepared to start on MST224.	Good! It would still be advisable to look at *Unit 1* as soon as possible, to check that there is nothing on which you want to spend extra time.

Do contact your Student Support Team via StudentHome if you have any queries about your suitability for the module.