An intersubjective model of agency for game theory

Vivienne Brown
The Open University, UK
v.w.brown@open.ac.uk

forthcoming Economics and Philosophy

Abstract
This paper proposes a new interpretation of noncooperative games that shows why the unilateralism of best-reply reasoning fails to capture the mutuality of strategic interdependence. Drawing on an intersubjective approach to theorising individual agency in shared context, including a non-individualistic model of common belief without infinite regress, the paper develops a general model of a $2 \times 2$ simultaneous one-shot noncooperative game and applies it to games including Hi-Lo, Stag Hunt, Prisoners’ Dilemma, Chicken, BoS and Matching Pennies. Results include High as the rational choice in Hi-Lo, and Cooperate as a possible rational choice in the Prisoners’ Dilemma.

Keywords: ‘each of us’, intersubjective reasoning, intersubjective expected payoff (ISEP) model, instrumental cooperation, resolvable coordination problems

1 Theorising agency for game theory
The paradigm model of agency in classical game theory is that of the rational maximizing individual. This model of agency has been variously finessed and challenged by theorists trying to explain the coordination and cooperation that are generally agreed to be essential to human and social life but which are not well explained by classical game theory. Two main approaches have been influential in arguing for a less individualistic understanding of agency for game theory. The social preferences approach argues for the importance of other-regarding preferences in explaining coordination and cooperation (e.g. Bolton and Ockenfels 2000; Fehr and Fischbacher 2002; Bicchieri 2006), and the team agency approach argues that if players
reason as members of a team this can facilitate coordination and cooperation (Sugden 1993, 2000; Bacharach 2006). In spite of the differences between these approaches, both aim to temper what they see as the excessive individualism of classical game theory by retheorising preferences even though this results in changing the game.

This paper proposes a new approach to noncooperative game theory by retheorising the way that individual players cognize the game, taking preferences as given and without assuming bounded rationality. The paper draws on a new intersubjective model of individual agency in shared social context, which construes agents’ cognizance of the situation in terms of the shared phenomenological standpoint of each of us (Brown 2019). This standpoint combines the singularity of agency together with agents' self-inclusion in the plurality ‘us’, thus dissolving the standard individualist – collectivist dichotomy. It also yields a non-individualistic model of common belief, without infinite regress, in terms of beliefs that ‘each of us’ has. Applied to game theory, players’ cognizance of their shared participation in a game is modelled in terms of true common beliefs about ‘each of us’, such as the true common belief that ‘each of us (players of game G) aims to maximize individual payoff’. This provides a non-individualistic approach that is consistent with the classical assumption of individual maximization (irrespective of whether preferences include social preferences), but it transforms the logic of individual choice and shows why the unilateralism of best-reply reasoning makes it too individualistic to capture the mutuality of strategic interdependence. The intersubjective model of agency thus provides new resources for analysing coordination and cooperation, whilst remaining within the classical assumption of individual maximization and without relying on preference change to generate new results.

Section 2 explains the intersubjective approach to theorising individual agents in shared social context, including a new model of common belief without infinite regress. Section 3 derives the intersubjective expected payoff (ISEP) model of a $2 \times 2$ simultaneous one-shot noncooperative game, with given preferences (which can include social preferences), which provides a new interpretation of rational choice as individual maximization of intersubjective expected payoff, together with a new solution concept and a new typology of noncooperative games. Section 4 analyses ISEP solutions and their Pareto properties in games across the typology; for example, High is the only possible rational choice in Hi-Lo, and Cooperate is a

---

1 This interpretation of ‘each of us’ is not the same as the individualist reading of ‘each of us’ as ‘I’ and ‘you’, which disregards the significance of the inclusive plurality, ‘us’.
possible rational choice in the Prisoners’ Dilemma. Section 5 concludes with a brief discussion.²

2 The intersubjective model of agency

2.1 Introducing the intersubjective stance³

A difficulty in theorising individual agency in shared social context is how to combine both the singularity and the shared social contextuality of individual agents. Approaches that focus on ‘I’ or on ‘we’ emphasize either the singularity or collectivity of agents but have problems in combining the two dimensions. The intersubjective approach resolves this difficulty by theorising agents in terms of the shared phenomenological standpoint of each of us. As the linguistic expression, ‘each of us’, is grammatically singular, agents are theorised as singular entities, yet each agent includes herself in the plurality, ‘us’, equivalently with each of the others. Individual agents in a shared situation are thus modelled as reflexively aware of themselves as one of ‘us’. The combination of singularity and inclusive plurality captures the distinctive way that individual human agents cognize their shared situation with others, and this holds independently of individual preferences or moral beliefs, and independently of whether the shared situation involves congruent or competing interests or a combination of congruent and competing interests. The intersubjective approach thus dissolves the individualist – collectivist dichotomy for individual agents in shared social context.⁴

‘Each of us’ is not the same as ‘every’ in universal quantification because it includes self-quantification. ‘Each of us is F’ is internal universal quantification which involves self-quantification by a subject who includes herself in ‘us’. By contrast, ‘every subject in A is F’ is external universal quantification by an external theorist or observer over items that are not theorized as having any cognizance of the quantification, or of set A, or of their own inclusion in set A.

The significance of this distinction for game theory is that the interactive model of shared belief (mutual belief and common belief) which is standardly used in game theory is based on external

² This paper owes a debt to Adam Smith’s Theory of Moral Sentiments (TMS) for highlighting the significance of intersubjectivity for individual human agency, although the argument of this paper is independent of TMS. Brown (2011) interprets the intersubjectivity of TMS as providing some resources for analysing cooperation in the prisoners’ dilemma.
³ Sections 2.1 – 2.2 draw on Brown (2019).
⁴ Application of the intersubjective approach proposed in this paper is not restricted to game theory as it applies in any shared situation.
universal quantification, as from the perspective of the external theorist or observer, and does not capture subjects’ self-understanding that a belief is shared amongst ‘us’. This explains why, even with only two subjects, the interactive model of common belief involves infinite strings of individual beliefs, such as ‘\(a_1\) believes that \(a_2\) believes that \(a_1\) believes that \(a_2\) believes that …’. Even if this is interpreted informally as ‘I believe that you believe that I believe that you believe that …’, the framing in terms of ‘I’ and ‘you’ excludes the inclusive plurality ‘us’. In spite of the ubiquity of common belief in everyday life, the infinite strings of individual beliefs in the interactive model presuppose an abstract notion of cognition in the double sense that it fails to capture human beings’ understanding of beliefs shared amongst ‘us’ and it presupposes cognitive capacities beyond what is humanly possible. In contrast, the intersubjective approach models common belief much more simply, without infinite regress, in terms of the understanding that shared beliefs are held by ‘each of us’.

Section 2.2 outlines the intersubjective model of shared belief (including common belief), and Section 2.3 offers a preliminary indication of the implications of this model for game theory.

2.2 Intersubjective model of shared and common belief

A key epistemic notion is intersubjective belief. An intersubjective belief that \(p\) is the belief of a subject in reference group, \(R\) (a class in the distributive sense), that ‘each of us (in \(R\)) has the same belief that \(p\)’:

An intersubjective belief (IB) that \(p\) is the belief of a subject who includes herself in reference group, \(R\), that ‘each of us (in \(R\)) has the same belief that \(p\)’. The belief content, ‘each of us (in \(R\)) has the same belief that \(p\)’, involves internal universal quantification. By including herself in reference group, \(R\), a subject understands a shared belief that \(p\) as a belief that ‘each of us’ has, not as a belief that ‘I’ have and ‘you1’ have and ‘you2’ have and … and ‘you\(_n\)’ have. It is also a feature of intersubjective beliefs that the IB content,

\[\text{An IB that } p \text{ is true iff every subject in } R \text{ has the same belief that } p. \text{ The truth of an IB is independent of whether } p \text{ is true: an IB might be true even if } p \text{ is false (every subject might have the same false belief that } p), \text{ or it may be false even if } p \text{ is true (it is not the case that every subject has the same true belief that } p).\]
p, can include reference to ‘each of us’; for example, if a subject in R has the IB that p where ‘p’ is ‘each of us (in R) is a player of game G’, she believes ‘each of us (in R) has the same belief that each of us (in R) is a player of game G’.  

If every subject in R has the same IB that p, this is plural intersubjective belief (plural IB) that p; that is, every subject in R has the same belief that ‘each of us (in R) has the same belief that p’. Intersubjective beliefs thus provide a more economical approach to modelling shared belief than the interactive model of mutual belief framed in terms of ‘I’ and ‘you’.

Intersubjective beliefs may be taken to a higher (finite) degree of intersubjectivity, where the degree of intersubjectivity is given by the number of occurrences of ‘each of us (in R) has the same belief that’, although for human subjects IBs are likely to be of low degree of intersubjectivity. The IB above is a first-degree IB. Second-degree IB that p is a subject’s belief that ‘each of us (in R) has the same IB that p’, which is equivalent to the belief that ‘each of us (in R) has the same belief that each of us (in R) has the same belief that p’.  

If every subject in R has the same second-degree IB that p, this is second-degree plural IB that p.

An IB does not logically require a higher-degree IB. This means that a first-degree IB that p does not logically require a second-degree IB that p. For example, subject S might believe that ‘each of us (in room X) has the same belief that p’, but this does not logically imply S’s belief that ‘each of us (in room X) has the same IB that p’. Perhaps S is the only person who believes that each person in the room has the same belief that p.

In some shared situations, however, termed ‘transparent situations’, true first-degree IB can lead to true second-degree IB via a valid inference. If in this transparent situation, every subject in R makes the same inference from a true first-degree IB that p to a true second-degree IB that p, this is intersubjective common belief (ICB) amongst subjects in R:

There is intersubjective common belief (ICB) that p amongst subjects in reference group, R, iff every subject in R makes the same inference from a true first-degree IB that p to a true second-degree IB that p.

---

10 Shared beliefs about difference of belief are also accommodated. For example, if ‘p’ is ‘each of us has a different belief about X’, the IB that p is the belief of a subject in R that ‘each of us (in R) has the same belief that each of us (in R) has a different belief about X’.

11 A second-degree IB that p is true iff every subject in R has the same true first-degree IB that p.
It is the possibility of this inference in a transparent situation that explains the phenomenological experience of openness and obviousness that is characteristic of common belief. ICB that $p$ is true or false depending on whether $p$ is true or false.\footnote{Although every subject’s belief that ‘if each of us has the same belief that $p$ then each of us has the same IB that $p’$ is true, $p$ itself might be true or false.}

The only way for the interactive approach to differentiate between mutual belief and common belief is to construe common belief in terms of an infinite hierarchy (or recursive system) of beliefs. By contrast, the intersubjective model does not need to rely on an infinite degree of IB to differentiate between plural IB and common belief. This explains why the intersubjective approach provides a more economical model of common belief. It is also more intuitive and more plausible as a model of shared beliefs because it draws on subjects’ self-inclusion in the plurality ‘us’ and makes only modest assumptions about subjects’ cognitive capacities.

2.3 Implications of the intersubjective model of agency for game theory

The intersubjective model of agency, including the intersubjective model of common belief, makes possible a non-individualistic epistemic foundation for game theory that presupposes neither unrealistic cognitive capacities nor bounded rationality.\footnote{Some approaches (e.g. level-k thinking, Crawford et al. 2013) simplify the interactive approach by adopting limitations on the level of belief. In contrast, the intersubjective approach provides a simpler model of common belief.} Crucially, the content, $p$, of any intersubjective belief might refer to ‘each of us’, as in the ICB that ‘each of us (players of game $G$) aims to maximize individual payoff’. This makes possible intersubjective reasoning, or reasoning in terms of ‘each of us’, which avoids the unilateralism of best-reply reasoning framed in terms of ‘I’ and ‘you’. Intersubjective reasoning is incompatible with an approach that asks what ‘I’ should do given what (‘I’ believe) ‘you’ do.

In the intersubjective model of noncooperative games developed in this paper, principles of play apply equivalently to ‘each of us’ but each player acts individually according to these principles in order to maximize individual payoff. The intersubjective model of agency thus offers a new approach to strategic reasoning although with no dilution of the classical assumption of individual maximization. For example, the intersubjective model shows that game theory’s difficulties with the equilibrium selection problem in even a simple game such as Hi-Lo, which common intuition suggests ought to have a unique solution of (High, High), derive from its epistemic individualism and reliance on best-reply reasoning.
Adopting the intersubjective model of agency opens up a new approach to analysing noncooperative games, in addition to the analytic simplicity achieved by eliminating infinite regress from common belief. Although this does not guarantee a remedy for resolving every coordination problem, it provides new resources for analysing coordination and cooperation, with new results for a number of games that have so far proved intractable. 

3 Intersubjective expected payoff (ISEP) model

3.1 Assumptions of the model

This section develops an intersubjective model of a $2 \times 2$ simultaneous one-shot noncooperative game, $G$, which analyses the way in which players cognize the game in terms of ‘each of us, players of game $G$’. The model develops a form of intersubjective reasoning as from the standpoint of ‘each of us, players of game $G$’. Without prejudging the different ways in which the intersubjective model of agency might be applied to noncooperative games, it is assumed that there is true ICB about the game as follows:

A(i): Players have true ICB that ‘each of us is a player of game $G$’ and there is true ICB about all aspects of game $G$ including possible actions and the payoff structure representing players’ preferences over possible outcomes (preferences can include social preferences);

A(ii): Players have true ICB that ‘each of us, players of game $G$, aims to maximize individual payoff and acts consistently to achieve this aim’;

A(iii): Players have true ICB that principles of play for maximizing individual payoff apply equivalently to ‘each of us, players of game $G$’;

A(iv): Players have true ICB that ‘each of us, players of game $G$, is a stranger to the co-player, with no means of communicating with the co-player, and without any other exogenous information relevant to the game’. (This assumption is later relaxed.)

3.2 Uncertainty for ‘each of us’

Although there is ICB about the characteristics of the game and players’ motivation, players are uncertain about the co-player’s choice of action. This suggests an analysis in terms of subjective probabilities over the co-player’s actions, but this immediately raises a question of how to model players’ beliefs about the co-player’s subjective probabilities. The question of

---

14 The distinction between ‘solving’ a game and ‘resolving’ a coordination problem is developed in Section 3.5.
how to model players’ beliefs about the co-player’s subjective probabilities, without introducing infinite regress, bounded rationality or arbitrary restrictions on the level of belief, is resolved in the intersubjective approach by modelling individual agency in terms of ‘each of us, players of game G’. Players understand uncertainty not as a problem ‘for me’, or ‘for me, thinking about you, thinking about me, … ’, but as a problem for ‘each of us’.

There is true ICB that ‘each of us’ is uncertain about the co-player’s action and that ‘each of us’ assigns a subjective probability (SP) distribution over the co-player’s actions. But if ‘each of us’ assigns an SP distribution over the co-player’s actions, then ‘each of us’ is having an SP distribution assigned over her own actions by the co-player. This implies true ICB that ‘each of us’ both assigns an SP distribution over the co-player’s actions and is having an SP distribution assigned over her own actions by the co-player. Intersubjective reasoning thus ensures that players take account of the co-player’s SP distribution over their own actions equivalently with their own SP distribution over the co-player’s actions.

If each player could combine her SP distribution over the co-player’s actions with the co-player’s (independent) SP distribution over her own actions, this would yield identical joint probability distributions over action profiles. As players do not know the co-player’s SP distribution over their own actions, the best they can do is to make a conjecture about it. Each player therefore not only assigns an SP distribution over the co-player’s actions but also makes a conjecture about the co-player’s SP distribution over her own actions. This yields for each player a conjectured joint probability distribution over action profiles. Players’ conjectured joint probability distributions over action profiles are intersubjective probability distributions, and the probability values over action profiles are intersubjective probabilities. There is true ICB that ‘each of us’ has intersubjective probabilities over individual action profiles, although players do not know the values of the co-player’s intersubjective probabilities.

Theoretically, intersubjective probabilities (ISPs) resolve the question of how to model players’ beliefs about the co-player’s SPs without introducing infinite regress, bounded rationality or arbitrary restrictions on the level of belief. But in resolving this question, ISPs introduce a subtle

---

15 Henceforth, ‘each of us’ stands for ‘each of us, players of game G’.
16 Mostly in what follows, the subject player is treated as grammatically female in contrast with the co-player who is treated as grammatically male, but sometimes both are treated as female for grammatical convenience.
17 This notion of intersubjective probability is different from that proposed in Gillies (2000: ch. 8) which is a group probability, or consensus probability of the members of a group, such that all the individual members of a group assign the same subjective probability that event, E, obtains.
change in the interpretation of SPs over actions. Standardly, a player’s SP over the co-player’s action, \( A \), is her degree of belief that the co-player chooses action, \( A \). As explained over the course of this section, intersubjective reasoning makes players’ choice of action dependent on their ISPs, given payoffs, and this means that a player’s choice of action is dependent not only on her SPs over the co-player’s actions but also, and equivalently, on her conjecture about the co-player’s SPs over her own actions. If an SP expresses a player’s degree of belief that the co-player chooses a particular action, this would imply that a player’s choice of action is dependent on her conjecture about the co-player’s degree of belief that she herself chooses that particular action, as well as on her SP that the co-player chooses a particular action. But in advance of deciding which action she ought to choose, a player can have no reason for any conjecture about the co-player’s degree of belief that she herself chooses one action rather than the other.\(^{18}\) An implication of this is that the co-player’s SP over the subject player’s action, \( A \), should not be interpreted as his degree of belief that the subject player actually chooses action, \( A \). Instead, the co-player’s SP over the subject player’s action, \( A \), is interpreted as his degree of belief that action, \( A \), is choiceworthy for her in the stipulated sense that action, \( A \), is \textit{prima facie} fit to be chosen by her. A player’s conjecture about the co-player’s SP over her own action, \( A \), is thus her conjecture about the co-player’s degree of belief that action, \( A \), is choiceworthy for her. As each player is a co-player to the other player, it follows that \textit{each player’s SPs over the other player’s actions, \( A \) and \( B \), express the player’s degree of belief as to which action (or neither) is choiceworthy for the other player}. An SP greater than .5 expresses a player’s belief that the action is choiceworthy; if neither action is believed to be choiceworthy, SPs of .5 are assigned over both \( A \) and \( B \).

Each player’s ISP distribution is given by the product of the player’s SP distribution and conjecture over the co-player’s SP distribution, where SPs express a player’s degree of belief that an action is choiceworthy for the co-player. As a joint probability distribution requires independence of events, the ISP distribution requires that an action’s being choiceworthy for one player does not affect the (conjectured) SP that the action is choiceworthy for the other player. If independence for subjective theories is interpreted informationally, as argued by Mongin (2019, esp. pp. 3, 16), this requirement is met by the fact that an action’s being choiceworthy for one player carries no information on the action’s being choiceworthy for the other player.

\(^{18}\) This is evident in the regress where a player’s choice of action depends on her belief as to which action the co-player chooses, which in turn is dependent on his belief as to which action she chooses, which in turn is dependent on her belief as to which action he chooses, ….
The distinction between actions being ‘choiceworthy’ and ‘actually chosen’ is developed and explained in the course of this section, and will be further explored in the analysis of different games in Section 4. A preliminary statement of the distinction is that players’ SPs and conjectures about the co-player’s SPs express players’ degrees of belief that actions are choiceworthy, whereas actual choice of action is based on expected payoffs to actions which are given by payoffs weighted by ISPs.

Players’ ISPs for a 2 × 2 game with actions A and B are shown in Figures 1 and 2.

Player 1’s ISPs over action profiles are shown in Figure 1. Player 1’s SPs that actions A and B are choiceworthy for Player 2 are given in the columns \(0 \leq P_1 \leq 1\). Player 1’s conjecture about Player 2’s SPs that A and B are choiceworthy for her (Player 1) are given in the rows \(0 \leq P_{2(1)} \leq 1\); this conjecture is the *subjective probability conjecture* (SP conjecture). The cells of the matrix, which are the products of these two probabilities, are Player 1’s ISPs over the action profiles. A player’s four ISPs sum to 1.

**Figure 1**  Player 1’s intersubjective probabilities (ISPs) over action profiles

<table>
<thead>
<tr>
<th>Pl. 1’s conjecture about Pl. 2’s SP that A is choiceworthy for Pl. 1 (P_{2(1)})</th>
<th>Pl. 1’s SP that A is choiceworthy for Pl. 2 (P_1)</th>
<th>Pl. 1’s SP that B is choiceworthy for Pl. 2 (1 - P_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pl. 2’s SP that A is choiceworthy for Pl. 1 (P_2)</td>
<td>(ISP_{1A} \equiv P_{2(1)}P_1)</td>
<td>(ISP_{1B} \equiv P_{2(1)}(1 - P_1))</td>
</tr>
<tr>
<td>Pl. 1’s conjecture about Pl. 2’s SP that B is choiceworthy for Pl. 1 (1 - P_{2(1)})</td>
<td>(ISP_{1B} \equiv (1 - P_{2(1)})P_1)</td>
<td>(ISP_{1B} \equiv (1 - P_{2(1)})(1 - P_1))</td>
</tr>
</tbody>
</table>

Reading down the left-hand column, Player 1’s ISPs are \(ISP_{1A} \equiv P_{2(1)}P_1\) and \(ISP_{1B} \equiv (1 - P_{2(1)})P_1\), and reading down the right-hand column Player 1’s ISPs are \(ISP_{1A} \equiv P_{2(1)}(1 - P_1)\) and \(ISP_{1B} \equiv (1 - P_{2(1)})(1 - P_1)\). Player 1’s ISP for the action profile \((A, B)\), for example, denoted as \(ISP_{1AB}\), is Player 1’s conjectured joint probability that A is deemed choiceworthy for her (Player 1) by Player 2 and B is choiceworthy for Player 2.
Player 2’s ISPs over action profiles are shown in Figure 2 where Player 2’s SPs that A and B are choiceworthy for Player 1 are given in the rows ($0 \leq P_2 \leq 1$), and Player 2’s conjecture about Player 1’s SPs that A and B are choiceworthy for him (Player 2) are given in the columns ($0 \leq P_{1(2)} \leq 1$).

**Figure 2** Player 2’s intersubjective probabilities (ISPs) over action profiles

<table>
<thead>
<tr>
<th>Pl. 2’s SP that A is choiceworthy for Pl. 1</th>
<th>Pl. 2’s conjecture about Pl. 1’s SP that A is choiceworthy for Pl. 2 $P_{1(2)}$</th>
<th>Pl. 2’s conjecture about Pl. 1’s SP that B is choiceworthy for Pl. 2 $1 - P_{1(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_2$</td>
<td>$ISP_{2A}^A \equiv P_2 P_{1(2)}$</td>
<td>$ISP_{2A}^B \equiv P_2 (1 - P_{1(2)})$</td>
</tr>
<tr>
<td>$1 - P_2$</td>
<td>$ISP_{2B}^A \equiv (1 - P_2) P_{1(2)}$</td>
<td>$ISP_{2B}^B \equiv (1 - P_2)(1 - P_{1(2)})$</td>
</tr>
</tbody>
</table>

Reading across the top row, Player 2’s ISPs are $ISP_{2A}^A \equiv P_2 P_{1(2)}$ and $ISP_{2A}^B \equiv P_2 (1 - P_{1(2)})$, and reading across the bottom row Player 2’s ISPs are $ISP_{2B}^A \equiv (1 - P_2) P_{1(2)}$ and $ISP_{2B}^B \equiv (1 - P_2)(1 - P_{1(2)})$. Player 2’s ISP for the action profile (A, B), for example, denoted as $ISP_{2A}^B$, is Player 2’s conjectured joint probability that A is choiceworthy for Player 1 and B is deemed choiceworthy for him (Player 2) by Player 1.

It is not assumed that players’ ISP distributions are identical (e.g. it is not assumed that $ISP_{1A}^B = ISP_{2A}^B$) or that $P_1 = P_2$.\(^{19}\) Although there is ICB that ‘each of us’ assigns SPs and SP conjectures, players do not know the values of the co-player’s SPs / SP conjectures and so do not know the co-player’s ISPs.

### 3.3 Expected payoff maximization and the ISEP solution concept

There is true ICB that ‘each of us’ aims to maximize individual expected payoff and acts consistently to achieve this aim (A(ii)). As intersubjective reasoners recognise uncertainty for

\(^{19}\) ISP distributions are identical iff conjectures are correct, i.e. $P_{2(1)} = P_2$; $P_{1(2)} = P_1$. This does not imply $P_1 = P_2$. 
‘each of us’, they make conjectures about the co-player’s SPs as well as assign their own SPs. Individual expected payoffs are thus individual payoffs weighted by ISPs.

A player’s expected payoff to an action profile is the player’s payoff for that profile weighted by the player’s ISP for that profile, and a player’s expected payoff to an action is the sum of the expected payoffs to the relevant action profiles. These expected payoffs (to profiles and to actions) are intersubjective expected payoffs (ISEPs). To differentiate between them, a player’s expected payoff to an action profile, is a profile-ISEP, and the expected payoff to an action, is an action-ISEP (or simply ‘ISEP’ where the meaning is clear). Players choose the action that maximizes action-ISEP but the ISEP associated with a profile is the profile-ISEP. This is explained as follows.

Figure 3 gives the bimatrix of profile-ISEPs for a $2 \times 2$ simultaneous one-shot game, with ISPs as given in Figures 1 and 2, and payoffs $w_i, x_i, y_i, z_i, (i = 1, 2)$. It is not assumed that $w_1 = w_2$, $x_1 = x_2$, and so forth.

**Figure 3** Bimatrix of profile-ISEPs

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>$w_1 \cdot ISP_{1A}$, $w_2 \cdot ISP_{2A}$</td>
<td>$x_1 \cdot ISP_{1B}$, $y_2 \cdot ISP_{2B}$</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>$y_1 \cdot ISP_{1B}$, $x_2 \cdot ISP_{2B}$</td>
<td>$z_1 \cdot ISP_{1B}$, $z_2 \cdot ISP_{2B}$</td>
</tr>
</tbody>
</table>

In Figure 3, Player 1’s profile-ISEP for $(A, A)$ is $w_1 \cdot ISP_{1A}$; Player 2’s profile-ISEP for $(A, A)$ is $w_2 \cdot ISP_{2A}$; and so forth. These profile-ISEPs are denoted as $ISEP_{1A}, ISEP_{2A}$, and so forth. That is, $ISEP_{1A} = w_1 \cdot ISP_{1A}, ISEP_{2A} = w_2 \cdot ISP_{2A}$; $ISEP_{1B} = x_1 \cdot ISP_{1B}$; and so forth.

**Action-ISEPs** are given by the sum of the relevant profile-ISEPs; that is, an action’s expected payoff is the sum of the expected values of the relevant action profiles. For example, Player 1’s ISEP to action A (denoted $ISEP_{1A}$) is composed of the sum of the profile-ISEPs, $ISEP_{1A}$ and $ISEP_{1B}$. Players’ ISEPs for actions A and B are shown in (1):

\[
ISEP_{1A} \equiv ISEP_{1A} + ISEP_{1B} \equiv w_1 \cdot ISP_{1A} + x_1 \cdot ISP_{1B} \quad (1.1a)
\]

\[
ISEP_{1B} \equiv ISEP_{1B} + ISEP_{1B} \equiv y_1 \cdot ISP_{1B} + z_1 \cdot ISP_{1B} \quad (1.1b)
\]
Players choose the action that maximizes action-ISEP, given their payoffs and ISPs. For example, Player 1 chooses action $A$ iff $ISEP_1^A > ISEP_1^B$. If $ISEP_1^A = ISEP_1^B$, Player 1 just picks one of the two actions.

As explained in Section 3.2, each player’s SP over the co-player’s action, $A$, expresses her degree of belief that $A$ is choiceworthy for the co-player, not that $A$ is actually chosen by the co-player. This can now be made more precise: each player’s SP over the co-player’s action, $A$, expresses her degree of belief that $A$ is choiceworthy for the co-player, not that $A$ is ISEP-maximizing for the co-player, just as her conjecture about the co-player’s SP over her own action, $A$, is her conjecture about the co-player’s degree of belief that $A$ is choiceworthy for her, not that $A$ is ISEP-maximizing for her.

For each player, the beliefs that enter into the determination of which action (if either) is ISEP maximizing for her are thus her beliefs about whether actions are choiceworthy (i.e. her SPs and SPCs). There is no best-reply reasoning involved. As there is no best-reply reasoning in the ISEP model, there is no problem of potential regress.\(^{20}\)

The ISEP solution concept is given by action-ISEP maximization. A unique ISEP solution is the action profile comprising each player’s strongly ISEP-maximizing action. For example, in Figure 3 the action profile $(A, B)$ is the unique ISEP solution iff $ISEP_1^A > ISEP_1^B$ and $ISEP_2^A < ISEP_2^B$. The profile-ISEPs for ISEP solution $(A, B)$ are $(x_1 \cdot ISP_1^{AB}, y_2 \cdot ISP_2^{AB})$. A multiple ISEP solution is composed of action profiles comprising players’ weakly ISEP-maximizing actions. For example, in Figure 3 there is a multiple ISEP solution iff actions $A$ and $B$ have the same ISEP for at least one player. If actions are equally ISEP maximizing, the player just picks one of those actions.

ISEP solutions do not rely on beliefs about the action chosen by a co-player. According to the intersubjective approach, the unilateralism of best-reply reasoning makes it too individualistic to model the mutuality of strategic interdependence. It follows that best-reply solution concepts, including Nash equilibrium (NE), have no relevance for the ISEP model. Although an ISEP solution might coincide with a NE, there is no reason why it should; and even if an

\(^{20}\) Cf. Note 18.
ISEP solution does coincide with a NE, an action is chosen because it is ISEP maximizing, not because it is a Nash action. There is no equilibrium selection problem for the ISEP model.  

The rational choice of action is the strongly ISEP-maximizing action, if there is one; if actions have equal ISEPs, it is rational to just pick an action. There is true ICB that ‘each of us’ chooses the action that maximizes individual action-ISEP, or just picks an action if action-ISEPs are equal. SPs and SP conjectures are inputs into this process and express players’ degree of belief that particular actions are choiceworthy. As the criteria for identifying which actions might be deemed choiceworthy are independent of the criteria for actions being ISEP maximizing, it follows that actions deemed choiceworthy might not turn out to be ISEP maximizing.

3.4 Explaining SPs and SP conjectures: a new typology of noncooperative games

The question of how SPs and SP conjectures are explained relates to a long-standing debate whether game theorists should explain players’ SPs in rational or psychological terms (e.g. Kadane and Larkey 1982a,b; Harsanyi 1982a,b; Aumann 1987; Morris 1995; Larrouy and Lecouteux 2017). This debate, however, is premised on an individualistic approach and without distinction between ‘choiceworthy’ and ‘ISEP-maximizing’ actions. The ISEP model provides its own explanation of the assignment of SPs and SP conjectures according to principles that apply equivalently to ‘each of us’ (hereafter ‘SP conjectures’ are denoted ‘SPCs’). This intersubjective reasoning yields a new typology of noncooperative games.

It was argued in Section 3.2 that SPs / SPCs express degrees of belief that actions are choiceworthy. This gives the assignment rule governing SP / SPC values in a $2 \times 2$ game with actions $A$ and $B$: ‘each of us’ assigns SP / SPC greater than .5 over $A$ or $B$ according to which action is deemed choiceworthy; and if neither action is deemed choiceworthy, SPs / SPCs equal to .5 are assigned over both $A$ and $B$. In the absence of exogenous information relevant to the game (assumption A(iv)), there are two methods for identifying which actions might be deemed choiceworthy.

---

21 Nash’s theory of equilibrium points in $n$-person games (Nash 1950) was used in Arrow and Debreu’s (1954) proof of the existence (i.e. logical consistency) of general competitive equilibrium. But a foundational question that never seems to be asked is why the same equilibrium concept is appropriate in games where individual choice of action is subject to players’ mutually recognised interdependence and in a model where agents maximize subject to variables that are taken as exogenous for them individually. Ironically it is because Nash equilibrium does not capture the mutuality of interdependent maximization that it could be adopted for the proof of the existence of general competitive equilibrium.
One method for identifying which actions might be deemed choiceworthy is to rank action profiles in terms of payoffs for ‘each of us’. This gives the action-profile method for identifying which actions might be deemed choiceworthy: if there is a unique action profile that strongly Pareto-dominates at least one profile and is not itself weakly Pareto-dominated by any other profile, the actions comprising this ‘uniquely best’ profile are identified as the actions that might be deemed choiceworthy. The existence of this uniquely best profile is an indicator of a possible congruence of individual interests and hence the possibility of individually advantageous coordination. Keying into the possibility of individually advantageous coordination is thus provided by the intersubjective stance of ‘each of us’. The other method for identifying which actions might be deemed choiceworthy is to rank individual actions according to their average payoff. This gives the average-payoff method for identifying which actions might be deemed choiceworthy: if there are actions with a higher average payoff, these actions are identified as the actions that might be deemed choiceworthy.

Given the diversity of possible payoff structures, the action-profile method and average-payoff method are not necessarily in concordance in identifying which actions might be deemed choiceworthy. There are three categories:

1. Games where the action-profile and average-payoff methods are in concordance in identifying which actions might be deemed choiceworthy: there is a uniquely best action-profile and this profile comprises actions with the higher average payoff. Such actions are deemed choiceworthy. Implementation of the assignment rule requires SPs / SPCs greater than .5 over actions identified by both methods.

2. Games where the action-profile and average-payoff methods are not in concordance in identifying which actions might be deemed choiceworthy: there is a uniquely best profile but, for at least one of the players, the actions comprising this profile do not have a higher average payoff. Players exercise personal judgment as to which method to adopt for the player(s) whose payoffs give rise to non-concordance, except in symmetric games where the action-profile method is required if (i) adoption of the average-payoff method by ‘each of us’ would result in an action profile that is strongly Pareto-dominated by the uniquely best profile, or (ii) neither action is identified by the average-payoff method because actions have equal average payoffs.

---

22 Action profile \((X, Y)\) strongly Pareto-dominates profile \((x, y)\) iff each player’s payoff is greater for \((X, Y)\) than for \((x, y)\). Action profile \((X, Y)\) is weakly Pareto-dominated by \((x, y)\) iff at least one player’s payoff for \((x, y)\) is greater than for \((X, Y)\) and neither player’s payoff for \((x, y)\) is worse than for \((X, Y)\).
These two principles are not mandatory for asymmetric games because asymmetry of payoffs might introduce additional issues for considering which method to adopt.

Actions identified according to the above are deemed choiceworthy. Implementation of the assignment rule requires SPs / SPCs greater than .5 over the actions identified according to the above, with SPs of .5 assigned over both A and B if there seems no reason to adopt one method rather than the other and hence no reason to deem either action as choiceworthy.

3. Games where only the average-payoff method applies because there is no uniquely best profile: implementation of the assignment rule requires SPs / SPCs greater than .5 over the actions with higher average payoffs, as these are deemed choiceworthy, with SPs of .5 assigned over A and B if the actions have equal average payoffs, as neither is deemed choiceworthy.

These categories provide a new typology of 2 × 2 noncooperative games:

1. Games with concordance of action-profile and average-payoff methods
2. Games with non-concordance of action-profile and average-payoff methods
3. Games where only the average-payoff method applies.

In each of these categories of games, players assign SPs / SPCs in accordance with the assignment rule: SPs / SPCs greater than .5 are assigned over A or B according to which action is deemed choiceworthy; and if neither action is deemed choiceworthy, players assign SPs / SPCs of .5 over both A and B. There is true ICB about the assignment rule and the two methods of implementation, although there is no ICB about the particular values of SPs / SPCs as players do not know the values assigned by the co-player.

In assigning SPs / SPCs, players are required to exercise personal judgment in two ways, viz. in assigning particular values of SPs / SPCs within the prescribed range, and, in some cases of non-concordance of methods, in deciding which (if either) method to apply. Exercise of personal judgment might thus influence which action turns out to be individually ISEP-maximizing. The implications of this will be explored across the typology of games in Section 4.

If the assumption excluding exogenous information (assumption A(iv)) is relaxed, players use exogenous information in whatever ways might be helpful in identifying which actions might be deemed choiceworthy. SPs / SPCs are assigned in accordance with the assignment rule. This is developed in Sections 3.5 and 4.3.
The ISEP model provides an application of intersubjective reasoning to game situations such that the rational choice of action is the player’s ISEP-maximizing action; or, if actions have equal ISEPs, it is rational to just pick an action. It is thus possible for rational choice (or pick) of action to depend on a player’s exercise of personal judgment.

3.5 ISEP solutions and Pareto efficiency

It was noted in Section 2.3 that the intersubjective approach to analysing noncooperative games does not guarantee a remedy for resolving every coordination problem. This can now be made more precise for the ISEP model by differentiating between solving a game, and resolving or dissolving a coordination problem.

Solving a game requires finding the ISEP solution (unique or multiple). As every game has an ISEP solution, there is no game that cannot be solved according to the ISEP model. This is consistent with the argument above (Section 3.3) that there is no equilibrium selection problem for the ISEP model. But resolving a coordination problem requires that the ISEP solution of the game is Pareto efficient, given only endogenous information. A unique ISEP solution is Pareto efficient iff there is no other action profile with a higher profile-ISEP for one player and at least as good a profile-ISEP for the other player; and a multiple ISEP solution is Pareto efficient iff for every action profile there is no other action profile with a higher profile-ISEP for one player and at least as good a profile-ISEP for the other player. Thus a unique ISEP solution resolves the coordination problem iff, given only endogenous information, the action profile comprising it is Pareto efficient in profile-ISEPs; and a multiple ISEP solution resolves the coordination problem iff, given only endogenous information, every action profile is Pareto efficient in profile-ISEPs, so that whichever actions are picked, the resulting action profile is Pareto efficient.

For a coordination problem to be resolvable, it is required only that there is a possible Pareto-efficient ISEP solution for that game (given only endogenous information); it is not required that every possible ISEP solution for that game is Pareto efficient. If, however, it is not possible for any ISEP solution to be Pareto efficient (given only endogenous information), the coordination problem is irresolvable.

If the assumption excluding exogenous information is relaxed, a coordination problem is dissolved if exogenous information enables players to identify actions deemed choiceworthy and hence assign SPs / SPCs that result in a Pareto-efficient ISEP solution. Thus, consistently
with Schelling’s argument (1960/1980), it is possible for even an irresolvable coordination problem to be dissolved if there is exogenous information available to players.

In games where communication is possible, such communication has effect (if it does) by influencing players’ SPs / SPCs. There is therefore no presumption that communication or agreement amounts only to ‘cheap talk’. Communication or agreement is ‘rich talk’ if it results in an ISEP solution that benefits both players. Communication or fake agreement aimed at deceiving the co-player is ‘false talk’.

3.6 The ISEP model

This section has argued that the ISEP model applies intersubjective reasoning to noncooperative games. This might be summarised intuitively along the following lines: It is common belief that ‘each of us’ aims to maximize individual payoff but ‘each of us’ is uncertain which action the other player will choose (or pick), so the best thing for ‘each of us’ to do individually is to assign SPs and SPCs according to which actions are deemed choiceworthy, and then choose the action that has greater expected payoff (or just pick an action if the actions have equal expected payoffs).

The ISEP model is a general model that applies to all $2 \times 2$ simultaneous one-shot noncooperative games, including pure coordination games, dilemma games and zero-sum games. Section 4 illustrates the range of application of the ISEP model across the typology, together with the welfare Pareto properties of ISEP solutions to different games.

4 Application of the ISEP model across the typology of noncooperative games

4.1 Resolvable coordination problems: symmetric games

4.1.1 Type 1: Concordance of action-profile and average-payoff methods

Hi-Lo, shown in Figure 4, is one of the simplest examples of the equilibrium selection problem. According to the standard analysis, there are two pure-strategy NE, $(High, High)$ and $(Low, Low)$, such that $(High, High)$ strongly Pareto-dominates $(Low, Low)$. Game theorists accept that it is common intuition that players ought to choose High, but there is no rationale for this intuition according to standard game theory.23

---

23 Player 1 ought to choose $High (Low)$ iff she believes that Player 2 chooses $High (Low)$, but Player 2 chooses $High (Low)$ iff he believes that Player 1 chooses $High (Low)$, … . Cf. Note 18.
According to the intersubjective approach, there is true ICB that ‘each of us chooses High or Low’. Given the simplicity of Hi-Lo, it is intuitively obvious that ‘each of us ought to choose High’. For each player individually, if ‘each of us ought to choose High’, it follows that ‘I ought to choose High’ because ‘I’ am one of ‘us’. Each player individually understands that she herself ought to choose High, and there is true ICB that ‘each of us’ understands this. NE is irrelevant. There is no equilibrium selection problem.

This intuitive understanding of Hi-Lo is supported by the ISEP model. Player i’s ISEPs for High and Low in Figure 4 are shown in (2) where j is the co-player:

\[
\begin{align*}
\text{ISEP}_i^H &= h \cdot ISP_i^{HH} = h \cdot P_i P_{j(0)} \\
\text{ISEP}_i^L &= l \cdot ISP_i^{LL} = l \cdot (1 - P_i)(1 - P_{j(0)}).
\end{align*}
\]

High is ISEP-maximizing for Player i iff \(\text{ISEP}_i^H > \text{ISEP}_i^L\).

As (High, High) is the uniquely best profile and High is identified by the average-payoff method, Player i’s SPs / SPCs for High are required to be greater than .5 (that is, .5 < \(P_i\) ≤ 1; .5 < \(P_{j(0)}\) ≤ 1). This implies \(ISP_i^{HH} > ISP_i^{LL}\). Given \(h > l\), it follows that \(\text{ISEP}_i^H > \text{ISEP}_i^L\).

High is ISEP-maximizing and the only possible rational choice. The ISEP solution is (High, High). As this solution is Pareto efficient in profile-ISEPs (further, it strongly Pareto-dominates all other profiles), it resolves the coordination problem in Hi-Lo.

4.1.2 Type 2: Non-concordance of action-profile and average-payoff methods

---

24 Figure 4a. Bimatrix of profile-ISEPs for Hi-Lo
1. Stag Hunt

In Stag Hunt, \( w > y \geq z > x \) (in the notation of Figure 3), as illustrated in Figure 5. The standard analysis is that Stag Hunt is an example of the equilibrium selection problem as there are two pure-strategy NE: \((\text{Stag, Stag})\) which is payoff dominant and \((\text{Hare, Hare})\) which is risk dominant.

Figure 5  Stag Hunt

According to the ISEP model, \((\text{Stag, Stag})\) is the uniquely best profile but \(\text{Hare}\) is identified by the average-payoff method. As adoption of the average-payoff method by ‘each of us’ implies \((\text{Hare, Hare})\) which is strongly Pareto-dominated by \((\text{Stag, Stag})\), the action-profile method is required with SPs / SPCs over Stag greater than .5.

Player \(i\)’s ISEPs for \(\text{Stag}\) and \(\text{Hare}\) in Figure 5 are shown in (3):

\[
\begin{align*}
ISEP_i^S &= 6 \cdot ISP_i^{SS} + 2 \cdot ISP_i^{SH} \\
ISEP_i^H &= 5 \cdot ISP_i^{HS} + 4 \cdot ISP_i^{HH}.
\end{align*}
\]

(3a)  (3b)

It is possible for either \(\text{Stag}\) or \(\text{Hare}\) to be ISEP maximizing.

SPs / SPCs over \(\text{Stag}\) only incrementally above .5 are not necessarily sufficient for \(\text{Stag}\) to be ISEP maximizing, just as high values of SPs / SPCs over \(\text{Stag}\) are not necessarily required. For example, in Figure 5, if Player \(i\)’s SPs and SPCs are both proportionate to relative payoffs in the profiles \((S, S)\) and \((H, H)\), that is, if \(P_i = P_{j(i)} = .6\), this is sufficient for \(\text{Stag}\) to be ISEP maximizing.\(^{25}\) If both players assign SPs and SPCs of .6, the ISEP solution is \((\text{Stag, Stag})\)

\(^{25}\) In Stag Hunt in Figure 5, if \(P_i = P_{j(i)} = .6:\)

\[
\begin{align*}
ISEP_i^S &= 6 \cdot .36 + 2 \cdot .24 = 2.64 \\
ISEP_i^H &= 5 \cdot .24 + 4 \cdot .16 = 1.84.
\end{align*}
\]

In this and other numerical examples, equal SPs and SPCs are adopted for simplicity.
which is Pareto efficient and resolves the coordination problem. The only Pareto-efficient solution in Stag Hunt is \((\text{Stag, Stag})\).

The standard view is that the issue for Stag Hunt is the degree of ‘trust’ that players have that the co-player chooses \(\text{Stag}\). What is significant in the ISEP model is players’ ‘trust’ that ‘each of us’ deems \(\text{Stag}\) to be sufficiently choiceworthy for the co-player. This is registered in players’ ISPs. For example, if \(P_i = P_{j(i)} > .52\) in Stag Hunt in Figure 5, this registers just enough ‘trust’ for \(\text{Stag}\) to be ISEP maximizing.

2. Prisoners’ Dilemma

The Prisoners’ Dilemma (PD) is arguably the most controversial of all games. Although the non-cooperative solution is defended by standard game theory (most comprehensively in Binmore 1994), it has raised a huge experimental and interdisciplinary debate about the alleged ‘paradox of rationality’ according to which individual rationality conflicts with collective rationality (e.g. Rapoport 1988; Sally 1995; Ostrom 1998; Peterson 2015). Experimental results, which have been interpreted as suggesting cooperation of about 50% in the one-shot game, have prompted a range of interpretations, from emphasis on incompatibility with standard game theory, to emphasis on potential compatibility if inexperienced subjects are given time to learn. Influential explanations of one-shot cooperation that construe it as a significant social phenomenon have tended to analyse it either in terms of social preferences (e.g. Rabin 1993; Bolton and Ockenfels 2000; Fehr and Fischbacher 2002; Bicchieri 2006;) or in terms of team reasoning (Sugden 1993, 2000; Bacharach 2006), but both these approaches explain cooperation by changing the game. Recent contributions have offered explanations of cooperation in terms of: agreed Pareto optimization (Gauthier 2013), the Berge behaviour rule of self-interested mutual support (Courtois et al. 2015), team reasoning about mutual advantage (Sugden 2015; Karpus & Radzvilas 2018), and a simulationist theory of mindreading (Larrouy

26 If \(P_i = P_{2(1)} = P_2 = P_{1(2)} = .6\) in Stag Hunt in Figure 5, profile-ISEPs are \((2.16, 2.16)\) for \((S, S)\), \((1.2, .48)\) for \((H, S)\), and \((.64, .64)\) for \((H, H)\).

27 Stag Hunt: \(1. \ ISP^S > ISP^{SH} > ISP^{HH}, ISP^S > ISP^{HS} > ISP^{HH}\)
\(2. \ w > y \geq z > x\)
\(3. \ w \cdot ISP^S \) is greater than every other profile-ISEP.

28 In Stag Hunt in Figure 5, if \(P_i = P_{j(i)} = P:\)
\(ISEP_i^S = 6P^2 + 2P(1 - P)\)
\(ISEP_i^H = 5P(1 - P) + 4(1 - P)^2\)
\(\therefore ISEP_i^S > ISEP_i^H\) if \(P > .52\) (2 dp).
& Lecouteux 2017). The ISEP model analyses the PD in the same way as any other noncooperative game.

The simultaneous one-shot PD is illustrated in Figure 6 with actions Cooperate (C) and Not cooperate (N), and payoff values of 4 for ‘Temptation’, 3 for ‘Reward’, 2 for ‘Defect’ and 1 for ‘Sucker’ ($T > R > D > S; \frac{T+S}{2} < R$).

Figure 6  Prisoners’ Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>3, 3</td>
</tr>
<tr>
<td>N</td>
<td>4, 1</td>
</tr>
</tbody>
</table>

According to the standard analysis, Not cooperate is strictly dominant (it is better whichever action is chosen by the co-player). Assuming causal independence of actions, it is never rational to cooperate in a one-shot PD: $(N, N)$ is a Dominant Strategy Equilibrium, also unique strict NE, although it is strongly Pareto-dominated by $(C, C)$. As the dominance argument holds irrespective of whether a player knows the co-player’s payoffs, Not cooperate is the rational choice for a single decision-maker as well as for a game with two players.

In the intersubjective approach it makes a difference whether there is a single decision-maker or two players. The dominance argument holds for a single decision-maker but not for players of a game. $(C, C)$ is the uniquely best profile but $N$ has higher average payoffs. As $(N, N)$ is strongly Pareto-dominated by $(C, C)$, the action-profile method is required with SPs / SPCs over Cooperate greater than .5. It is possible for either Cooperate or Not cooperate to be ISEP-maximizing.

Greater confidence that Cooperate is choiceworthy makes it more likely that Cooperate is ISEP-maximizing. As with Stag Hunt, SPs / SPCs for Cooperate that are only incrementally above .5 are not necessarily sufficient for Cooperate to be ISEP maximizing, just as high SPs / SPCs for Cooperate are not necessarily required, although in general higher SPs / SPCs are required for Cooperate in the PD than for Stag in Stag Hunt because Reward < Temptation. For example, if Player $i$’s SPs and SPCs for Cooperate are proportionate to relative payoffs in
the profiles \((C, C)\) and \((N, N)\) in Figure 6, that is, if \(P_i = P_j(i) = .6\), this is only just sufficient for Cooperator to be ISEP maximizing.\(^{29}\) If both players assign SPs / SPCs of .6, the ISEP solution is \((C, C)\) which is Pareto efficient and so resolves the coordination problem.\(^{30}\) With SPs / SPCs over Cooperator greater than .5, \((C, C)\) is Pareto efficient, \((N, N)\) is Pareto inefficient, and \((C, N)\) and \((N, C)\) can be Pareto efficient or Pareto inefficient.\(^{31}\)

The ISEP model explains the possibility of rational cooperation in the PD without reliance on change in preferences, bounded rationality, team reasoning or any other ad hoc argument. The ISEP solution is dependent on particular payoff values as well as on the particular values of SPs / SPCs greater than .5. A PD cooperative solution is more (less) likely with higher (lower) payoffs for Reward and Sucker relative to Temptation and Defect.

Cooperation by ISEP-maximizers is instrumental cooperation. In contrast with explanations of one-shot cooperation in terms of altruism, morality or mutual benefit, the ISEP model shows that cooperation in the PD (as in Stag Hunt) is consistent with individual maximization. It follows that cooperation per se is not prosocial. Whether cooperation is deemed prosocial or antisocial depends on a normative judgment about the action, a judgment that stands outside game theory. If residents of a street stop littering the pavement, this is an example of prosocial cooperation; gangsters denying a crime they have committed together in order to avoid merited punishment and corporate collusion to defraud the public are examples of antisocial cooperation.

### 4.1.3 Type 3: Only average-payoff method applies

Payoffs for Chicken are given by \(y > w > x > z\). Chicken is illustrated in Figure 7 where \(w + x > y + z\) (e.g. van Basshuysen 2017: 158). According to the standard analysis, there are two pure-strategy NE, \((\text{Swerve, Straight})\) and \((\text{Straight, Swerve})\).

\(^{29}\) In the PD in Figure 6, if \(P_1 = P_2 = P\):
\[
\text{ISEP}_C^i = 3P^2 + P(1 - P)
\]
\[
\text{ISEP}_N^i = 4P(1 - P) + 2(1 - P)^2
\]
\[\therefore \text{ISEP}_C^i > \text{ISEP}_N^i \text{ if } P > .59 \text{ (2 dp).}\]

\(^{30}\) If \(P_1 = P_2 = P_3 = P_4 = .6\) in the PD in Figure 6, profile-ISEPs are \((1.08, 1.08)\) for \((C, C)\), \((.96, .24)\) for \((N, C)\), and \((.32, .32)\) for \((N, N)\).

\(^{31}\) PD:
1. \(\text{ISP}_{CC} > \text{ISP}_{CN} > \text{ISP}_{NN}; \text{ ISP}_{CC} > \text{ISP}_{NC} > \text{ISP}_{NN}\)
2. \(T > R > D > S\)
3. \(\therefore R \cdot \text{ISP}_{CC} > D \cdot \text{ISP}_{NN}; R \cdot \text{ISP}_{CC} > S \cdot \text{ISP}_{CN}; R \cdot \text{ISP}_{CC} \geq T \cdot \text{ISP}_{NC}\)
4. \(\therefore (C, C)\) is Pareto efficient, \((N, N)\) is Pareto inefficient, and \((N, C)\) and \((C, N)\) can be Pareto efficient or Pareto inefficient.
Figure 7 Chicken

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Swerve</strong></td>
<td><strong>Straight</strong></td>
</tr>
<tr>
<td>Player 1</td>
<td>0, 0</td>
<td>-1, 1</td>
</tr>
<tr>
<td></td>
<td>1, -1</td>
<td>-10, -10</td>
</tr>
</tbody>
</table>

There is no uniquely best profile in Chicken. If Swerve has higher average payoffs than Straight, as in Figure 7, SP / SPCs over Swerve are required to be greater than .5. For SP / SPCs over Swerve that are not in the higher part of the range (e.g. \( P_i = P_{j(i)} < .83 \)), the ISEP-maximizing action is Swerve, but for SPs / SPCs in the higher part of the range (e.g. \( .83 < P_i = P_{j(i)} < 1 \)), the ISEP-maximizing action is Straight.\(^{32}\) Intuitively, confidence that Swerve is choiceworthy for the co-player who is conjectured to be confident that Swerve is choiceworthy for the subject player makes Straight ISEP maximizing. In Figure 7, ISEP solutions \((Sw, Sw)\), \((Sw, St)\) and \((St, Sw)\) are Pareto efficient, and \((St, St)\) is Pareto inefficient.\(^{33}\)

This analysis changes if Straight has the higher average payoff, that is, if \( w + x < y + z \). For example, if \( y = 10 \) in Figure 7 with other payoffs unchanged, SPs / SPCs over Straight are required to be greater than .5. If \( 1 - P_i = 1 - P_{j(i)} > .52 \), Swerve is ISEP maximizing.\(^{34}\) Intuitively, confidence that Straight is choiceworthy for the co-player who is conjectured to be confident that Straight is choiceworthy for the subject player makes Swerve ISEP maximizing.

\(^{32}\) In Chicken in Figure 7, if \( P_i = P_{j(i)} = P \):
\[
ISEP_i^{Sw} = -P(1 - P) \\
ISEP_i^{St} = P(1 - P) - 10(1 - P)^2 \\
\therefore ISEP_i^{Sw} > ISEP_i^{St} \text{ if } P < .83 ; ISEP_i^{Sw} < ISEP_i^{St} \text{ if } .83 < P < 1 ; ISEP_i^{Sw} = ISEP_i^{St} \text{ if } P = .83, 1 \text{ (2 dp.)}
\]

\(^{33}\) Chicken as in Figure 7:

1. Figure 7a. Bimatrix of profile-ISEPs

\[
\begin{array}{cc}
0, 0 & -P(1 - P) \\
0, -P(1 - P) & -10(1 - P)^2
\end{array}
\]

2. \( ISP_i^{StSw} > ISP_i^{StSt} > ISP_i^{SwSt} \); \( ISP_i^{SwSw} > ISP_i^{StSw} > ISP_i^{StSt} \)

3. \( (Sw, Sw), (St, Sw) \) and \( (Sw, St) \) are Pareto efficient, and \( (St, St) \) is Pareto inefficient.

\(^{34}\) In Chicken in Figure 7 with \( y = 10 \), if \( P_i = P_{j(i)} = P \):
\[
ISEP_i^{Sw} = -P(1 - P) \\
ISEP_i^{St} = 10P(1 - P) - 10(1 - P)^2 \\
\therefore ISEP_i^{Sw} > ISEP_i^{St} \text{ if } P < .48 \text{ (2 dp.)}
\]
for the subject player. In Figure 7 with $y = 10$, $(Sw, Sw)$, $(Sw, St)$ and $(St, Sw)$ are Pareto efficient. The coordination problem is resolvable in both versions of Chicken.

4.2 Resolvable coordination problems: asymmetric games

4.2.1 Type 1: Concordance of action-profile and average-payoff methods

An asymmetric example of a Type 1 game is Escape (Crawford et al. 2013: 25–27), shown in Figure 8. Ash has two possible routes of escape from his pursuers: travel south (kinder climate) or north (harsh climate). In the standard analysis there are no pure-strategy NE.

**Figure 8 Escape**

<table>
<thead>
<tr>
<th></th>
<th>Ash’s pursuers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>South</td>
</tr>
<tr>
<td>Ash</td>
<td></td>
</tr>
<tr>
<td>South</td>
<td>−1, 3</td>
</tr>
<tr>
<td>North</td>
<td>0, 1</td>
</tr>
</tbody>
</table>

In Escape $(South, South)$ is the uniquely best profile and $South$ has higher average payoffs. SPs / SPCs over $South$ are required to be greater than .5 for both players. For the pursuers this makes $South$ ISEP-maximizing.

The situation is more complicated for Ash as high confidence that $South$ is choiceworthy for the pursuers makes $North$ ISEP maximizing for him. For example, if Ash assigns $SP = SPC$ over $South$ greater than $\frac{2}{3}$ or if he assigns $SP = 1$ that $South$ is choiceworthy for his pursuers,

\[ISP_P^{SS} > ISP_P^{SN} > ISP_P^{NN}\] implies \((3 \cdot ISP_P^{SS} + ISP_P^{NS}) > 2 \cdot ISP_P^{NN}\).
North is ISEP maximizing for him. The possible ISEP solutions, (South, South) and (North, South), are both Pareto efficient and so either resolves the coordination problem.

4.2.2 Type 2: Non-concordance of action-profile and average-payoff methods

Howard Raiffa (1992: 171) puzzles over a game, shown as ‘Raiffa’s Puzzle’ in Figure 9, with pure-strategy NE (B, A) and (A, B), such that (B, A) strongly Pareto-dominates all other profiles and (A, B) strongly Pareto-dominates the two remaining profiles, but A has a large negative payoff for Player 2 if Player 1 chooses A. Raiffa opts for (B, A) as the solution but is not at ease with this.

**Figure 9  Raiffa’s Puzzle**

<table>
<thead>
<tr>
<th>Player 2</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0, -1000</td>
<td>10, 8</td>
</tr>
<tr>
<td>B</td>
<td>12, 10</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

The action-profile method identifies (B, A) as the uniquely best profile in Raiffa’s Puzzle. For Player 1’s actions, this is in concordance with the average-payoff method, requiring SPs / SPCs greater than .5 that B is choiceworthy for her (i.e. $P_2$ and $P_{2(1)}$ are both less than .5). For Player 2’s actions, the two methods are not in concordance as the average-payoff method identifies B. Player 2’s large negative payoff to A for the action-profile (A, A) provides good reason for players to adopt the average-payoff method for Player 2’s actions, which implies that Player 1

---

38 If Ash assigns $P_A = P_{p(A)} = P$

$ISEP_A^S = -P^2 + P(1 - P)$

$ISEP_A^N = -2(1 - P)^2$

∴ $ISEP_A^S < ISEP_A^N$ if $P > \frac{2}{3}$.

If Ash assigns $P_A = 1$ and $P_{p(A)} > .5$:

$ISEP_A^S = -P_{p(A)}$

$ISEP_A^N = 0$

∴ $ISEP_A^S < ISEP_A^N$.

39 Escape:

1. Figure 8a. Bimatrix of profile-ISEPs:

<table>
<thead>
<tr>
<th>-ISP^SS</th>
<th>3 ISP^SS</th>
<th>ISP^SN</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ISP^NN^S</td>
<td>-2 ISP^NN</td>
<td>0</td>
</tr>
</tbody>
</table>

2. ISP^SS > ISP^SN > ISP^NN; ISP^SS > ISP^NS > ISP^NN

3. ∴ (S, S), (N, S) and (S, N) are Pareto efficient, and (N, N) is Pareto inefficient.
assigns very low or zero SP that A is choiceworthy for Player 2 and Player 2 conjectures that Player 1 assigns very low or zero SP that A is choiceworthy for him (i.e. \( P_1 \) and \( P_{1(2)} \) are both very low or zero).

Player 1’s ISEPs are shown in (5):

\[
ISEP_1^A = 10 \cdot ISP_{1AB} \equiv 10P_{2(1)}(1 - P_1) \quad (5a)
\]
\[
ISEP_1^B = 12 \cdot ISP_{1BA} \equiv 12(1 - P_{2(1)})P_1 \quad (5b)
\]

As \( P_{2(1)} \) is less than \(.5\), and as \( P_1 \) is very low or zero, this implies \( ISEP_1^A > ISEP_1^B \). Player 1 chooses A. Even though B is deemed choiceworthy for Player 1, her very high SP that B is choiceworthy for Player 2 implies that her ISP for \((A, B)\) is so much greater than her ISP for \((B, A)\) that her ISEP to A is greater than to B.

Player 2’s large negative payoff to A for the profile \((A, A)\) implies \( ISEP_2^A < ISEP_2^B \) given that \( P_2 < .5 \) and \( P_{1(2)} \) is very low or zero. So Player 2 chooses B.

The ISEP solution for Raiffa’s Puzzle is \((A, B)\). As this ISEP solution is Pareto efficient (further, it Pareto-dominates all other profiles), it resolves the coordination problem.\(^{40}\) According to the ISEP model, Raiffa was right to be uneasy with \((B, A)\) as the solution.\(^{41}\)

4.2.3 Type 3: Only average-payoff method applies

1. Nash’s game

A game considered by John Nash (1951: 292, ex. 5) with two pure-strategy NE, \((A, A)\) and \((B, B)\), is shown in Figure 10. Nash notes that empirical tests ‘show a tendency toward’ \((A, A)\) (1951: 292).

Figure 10 Nash’s game

\[
\begin{array}{cc}
A & B \\
\hline
0 & -1000 \cdot ISP_{2AA} \\
12 \cdot ISP_{1BA} & 10 \cdot ISP_{1AB} \\
10 \cdot ISP_{2BA} & 0, 0
\end{array}
\]

\(^{40}\) Raiffa’s Puzzle:

1. Figure 9a. Bimatrix of profile-ISEPs

2. As \( P_1 \) and \( P_{1(2)} \) are very low or zero, \( ISP_{AA} \) and \( ISP_{BA} \) are very low or zero for both players

3. \( (A, B) \) is Pareto efficient (further, it Pareto-dominates all other profiles).

\(^{41}\) If, however, players agree to coordinate on \((B, A)\), such that \( P_1 \), \( P_{1(2)} \), \( 1 - P_2 \) and \( 1 - P_{2(1)} \) are sufficiently high to make \( ISEP_1^B > ISEP_1^A \) and \( ISEP_2^A > ISEP_2^B \), this provides an example of ‘rich talk’ as profile-ISEPs to both players are improved. (Best possible profile-ISEPs iff \( P_1 = P_{1(2)} = 1 - P_2 = 1 - P_{2(1)} = 1 \).)
There is no uniquely best profile in Nash’s Game. As the average-payoff method identifies A as having the higher payoff for each player, SPs / SPCs over A are required to be greater than .5 (i.e. $ISP^{AA} > ISP^{BB}$ for both players). This implies $ISEP^A > ISEP^B$ for both players, with ISEP solution (A, A). This resolves the coordination problem as (A, A) is Pareto efficient in profile-ISEPS. It also provides a possible explanation of the ‘tendency toward’ (A, A) noted by Nash.

2. Matching Pennies: zero-sum game

In Matching Pennies, shown in Figure 11, there are no pure-strategy NE.

![Figure 11 Matching Pennies](image)

The average-payoff method requires SPs / SPCs of .5 over both Head and Tail, which makes all ISPs equal to .25. As $ISEP^{H} = ISEP^{T} = .25 - .25 = 0$, players are indifferent between Head and Tail, and the multiple ISEP solution comprises all action profiles. It is rational to just pick an action. This multiple ISEP solution resolves the coordination problem because every action profile, which is either (.25, −.25 ) or (−.25, .25 ) in profile-ISEPs, is Pareto efficient.

4.3 Irresolvable coordination problems

The ISEP solutions in Sections 4.1 – 4.2 exclude exogenous information. If the assumption excluding exogenous information is relaxed, this opens up the possibility of an additional means for players to identify which actions might be deemed choiceworthy and hence assign SPs / SPCs in accordance with the assignment rule. If this results in a Pareto-efficient ISEP solution, the coordination problem is ‘dissolved’. Exogenous information thus makes it
possible to dissolve a coordination problem even in irresolvable games. The ISEP model thus interprets Schelling’s (1980/1960) argument in terms of the way that exogenous information might provide an additional means of identifying which actions might be deemed choiceworthy, hence making possible a Pareto-efficient solution even in irresolvable games.

1. Pure coordination game

A pure coordination game is shown in Figure 12. According to the standard analysis there are two pure-strategy NE, \((A, A)\) and \((B, B)\), with identical payoffs.

**Figure 12** Pure coordination game

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>1, 1</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>0, 0</td>
</tr>
</tbody>
</table>

The pure coordination game in Figure 12 is a symmetric Type 3 game. The average-payoff method requires SPs and SPCs of .5 for both \(A\) and \(B\), making all ISPs equal to .25. As \(\text{ISEP}^A = \text{ISEP}^B = .25\), players are indifferent between \(A\) and \(B\). The multiple ISEP solution comprises all action profiles, but as \((A, B)\) and \((B, A)\) are Pareto inefficient in profile-ISEPs, the coordination problem is irresolvable.

Coordination on \((A, A)\) or \((B, B)\) might be more likely if action labels are interpreted as carrying information that identifies which action might be deemed choiceworthy. For example, if there is ICB that ‘\(A\)’ takes priority over ‘\(B\)’ such that \(A\) is deemed choiceworthy, players assign SPs / SPCs greater than .5 over \(A\) in accordance with the assignment rule. This implies \(\text{ISP}^{AA} \succ \text{ISP}^{BB}\) for both players. In this case the ISEP solution, \((A, A)\), would be Pareto efficient (further, it would Pareto-dominate all other profiles), and so would dissolve the coordination problem.

2. Bach or Stravinsky (BoS)

In Bach or Stravinsky (BoS), shown in Figure 13, each player prefers coordination to non-coordination but each prefers coordination on a different action profile. According to the standard analysis there are two pure-strategy NE, \((B, B)\) and \((S, S)\):
Figure 13  Bach or Stravinsky (BoS)

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B</strong></td>
<td>2, 1</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>0, 0</td>
</tr>
</tbody>
</table>

BoS is an asymmetric Type 3 game. Applying the average-payoff method requires SPs / SPCs greater than .5 over B for Player 1 and over S for Player 2. This implies ISEP solution (B, S) which is Pareto inefficient in profile-ISEPs (it is Pareto-dominated by (B, B) and (S, S)). The coordination problem is irresolvable.

As widely discussed, relations of deference between the players might enable coordination. If there is ICB that Player 1 is deferential to Player 2 such that S is deemed choiceworthy, players assign SPs / SPCs greater than .5 over S in accordance with the assignment rule. This implies $ISP_{SS} > ISP_{BB}$ for both players. The ISEP solution is (S, S) iff $ISP_{1S} > 2 \cdot ISP_{1B}$. As this ISEP solution is Pareto efficient (further, it Pareto-dominates all other profiles), it dissolves the coordination problem.

5 Discussion

5.1 Philosophical rationale

This paper has argued that the intersubjective model of agency provides resources for a new model of 2 × 2 simultaneous one-shot noncooperative games. The ISEP model provides new results across a range of games and shows how coordination and cooperation are possible in games thought to be inconducive to such results. The ISEP model achieves this by differentiating between the intersubjectivity of the epistemic foundation, the subjectivity of players’ personal judgment in implementing the assignment rule, and the individuality of preference (including social preference) maximization.

The intersubjective model of agency dissolves what is widely seen as a dichotomy between the individual and the collective, and between individual reasoning and collective reasoning. This might raise a question whether the intersubjective model of agency relies on a particular theory of social cognition. For example, if the social framing of human mindedness makes the
intersubjective model of common belief more cognitively appropriate than the interactive model, perhaps this might suggest reliance on, or at least congruence with, a particular theory of mind or social cognition. Yet, the intersubjective model of shared belief is a conceptual model that does not rely on any particular theory to explain how human beings cognize that their beliefs are shared. It simply takes it for granted that human beings understand that some of their beliefs are shared amongst ‘us’.

Similarly the ISEP model does not take a stand on the psychology of personal judgment in implementing the assignment rule. For some games, for example Hi-Lo, application of the assignment rule results in a determinate ISEP solution. In these games the exercise of personal judgment is of no significance in determining the ISEP solution. For games where personal judgment does have significance for the ISEP solution, for example the PD, the ISEP model leaves it open as to how such personal judgment might be theorised or explained.

The ISEP model is thus parsimonious in its assumptions whilst leaving it as an open question whether theories of human psychology or social cognition might contribute to its further development. Furthermore, although the ISEP model is normative in explaining rational choice of action, it also illustrates the limits to a purely rationalist approach to game theory. It is also open to empirical testing; for example, by assessing the effects on choice of action if payoffs change within the parameters of a game (for example, the model predicts that Cooperate is more likely to be chosen in the PD if the payoff to Temptation is reduced). To the extent that the ISEP model provides support for ordinary human intuitions about some games, for example, that High should be chosen in Hi-Lo, it suggests that ordinary human reasoning might sometimes be more subtle, and also ‘smarter’, than is currently recognised by game theory.

5.2 Social norms

Although the ISEP model is consistent with the classical assumption of individual maximization, incorporating a role for players’ personal judgment opens up a new approach for analysing the role of exogenous information. This also has potential application to social norms and morality.

As players cognize the game in terms of ‘each of us’, the provenance of this ‘us’ may extend beyond the game to take in other identities, associations, presuppositions, shared norms, morality, and so forth. As intersubjective reasoners, players might thus draw, consciously or subconsciously, on a range of shared exogenous factors. This suggests that the boundary
between endogenous and exogenous information might in practice be permeable. This has potential applicability in explaining the influence of social norms and morality on rational choice of action in terms of their influence on players’ assessment of whether actions might be choiceworthy, independently of payoffs. In line with the argument made by some theorists that it is inappropriate to include social norms and morality in preferences (e.g. Sen 1977; Engelen 2017), the ISEP model offers a possible means of incorporating the independent influence of social norms and morality on rational choice of action in games.

Players might thus deem actions to be choiceworthy (and hence assign SPs / SPCs greater than .5) if these actions are seen as promoting coordination and cooperation in a society or social setting in which such behaviour is regarded favourably or accepted as normal. This might influence the ISEP solution in games such as Stag Hunt and the PD independently of payoffs. Social or group norms of various sorts, including malign norms as well as benign norms according to some (non-game-theoretic) notion of normativity, might thus be theorised as exerting an influence on rational action independently of payoffs.

5.3 Opening up a new approach

This paper has argued that the ISEP model provides a unified framework for analysing noncooperative games, without reliance on bounded rationality, altruism, social norms or morality, and without ad hoc responses for specific games. In this the ISEP model is consistent with the instrumentalist focus of classical game theory. But by theorising individual agency in terms of ‘each of us’, the ISEP model offers an alternative to epistemic individualism and a new understanding of coordination and cooperation. The ISEP model also offers the possibility of a new route for incorporating shared exogenous influences such as social norms and morality. The ISEP model thus opens up a new approach to game theory that might more accurately model the strategic reasoning of human players and better explain instances of the coordination and cooperation that are essential features of human and social life, as well as shedding new light on failures of coordination and cooperation.

Acknowledgements

Ancestors of parts of the paper were presented at: Philosophy Department, Open University, 2010, 2012; ‘Rethinking Social Economy’, CRASSH, Cambridge, 2010; CSMN, Oslo, 2010; Management School, Queen’s University Belfast, 2011; 15th SCEME Seminar in Tribute to
Mark Blaug, Glasgow, 2012; and 11th INEM conference, Erasmus University, Rotterdam, 2013. I thank participants at all these events, especially Bart Engelen, Renée Prendergast and Mauro Rossi. I also thank John Broome, James Forder, Derek Goldrei, James Konow, Michiru Nagatsu and Roberto Scanzieri for comments on ancestral drafts, and Andy Denis, Till Düppe and Tony Weidberg for early discussions. Initial work was carried out whilst I was an Academic Visitor at the Faculty of Philosophy, Oxford (2009–2010), and a Visiting Scholar at CSMN, Oslo (October 2010). I am hugely grateful to the anonymous referees for Economics and Philosophy for their extensive comments, questions and suggestions.

References


