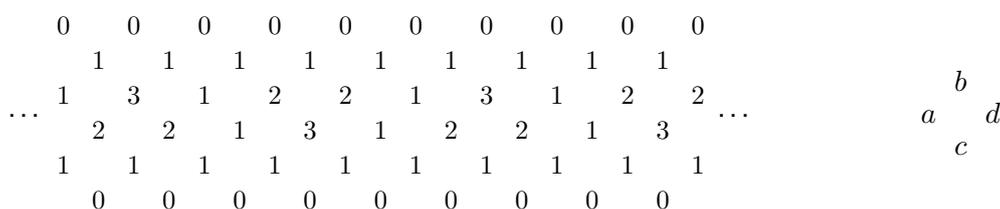


Project title	Frieze patterns and Farey complexes
Principal supervisor	Ian Short
Second supervisor	Katherine Staden
Discipline	Pure mathematics
Research area/keywords	Farey complex, frieze patterns, SL_2 -tilings
Suitable for	Full-time applicants, Part-time applicants

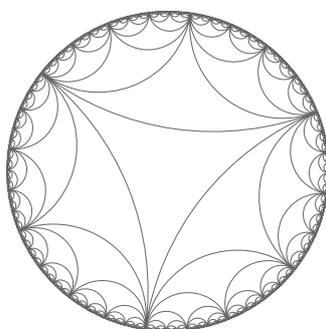
Project background and description

The genesis of this subject is in work of Conway and Coxeter from the 1970s on frieze patterns of positive integers. A frieze pattern is an array of integers of the type shown below on the left, with zeros at the top and bottom and positive integers in between, for which any diamond of four entries $a, b, c,$ and d satisfies the rule $ad - bc = 1$.



A positive integer frieze pattern (left) and a diamond of four entries (right)

Conway and Coxeter showed how to classify frieze patterns using polygons partitioned into triangles. More recently, Morier-Genoud, Ovsienko and Tabachnikov grounded this classification using the Farey complex (shown below), which is a tessellation of the hyperbolic plane by infinitely many triangles.



The Farey complex

The principal supervisor developed this approach in [3] and has an EPSRC grant to extend the work further still [4].

This project will look at frieze patterns over rings other than the integers. For smaller rings such as a finite ring, it is possible to construct an alternative Farey complex associated to the ring, which can be used to understand frieze patterns over that ring. These complexes are well known for the rings $\mathbb{Z}/n\mathbb{Z}$ but have not been studied in this context for other finite rings.

For larger rings the Farey complex is less useful and instead we will use tools from hyperbolic geometry.

A good starting point to enter the subject is the recent expository article [1]. For an overview, consult the survey article [2].

Background reading/references

- [1] Baur, Karin. Frieze patterns of integers. *The Mathematical Intelligencer* **43** (2021). <https://link.springer.com/article/10.1007/s00283-021-10065-x>
- [2] Morier-Genoud, Sophie. Coxeter's frieze patterns at the crossroads of algebra, geometry and combinatorics. *Bull. Lond. Math. Soc.* **47** (2015). <https://arxiv.org/abs/1503.05049>
- [3] Short, Ian. Classifying SL_2 -tilings. *To appear in Trans. Amer. Math. Soc.*. <https://arxiv.org/abs/1904.11900>
- [4] Short, Ian. The Farey framework for SL_2 -tilings. EPSRC Grant EP/W002817/1, £375,376 (commences July 2022). <https://gow.epsrc.ukri.org/NGBOViewGrant.aspx?GrantRef=EP/W002817/1>