School of Mathematics and Statistics
Faculty of Science, Technology, Engineering and Mathematics

2023 PhD Projects

Project title	The relational complexity of a finite permutation group
Principal supervisor	Dr Nick Gill
Second supervisor	Dr Bridget Webb
Discipline	Pure mathematics
Research area/keywords	group theory, permutation groups, homogeneity, relational structures
Suitable for	Full time or part time applicants

Project background and description

The notion of the relational complexity of a finite permutation group was first introduced by the model theorist Gregory Cherlin. He was interested in trying to understand the universe of finite permutation groups – it turns out that if one organises that universe using relational complexity, then one can somehow understand different kinds of sporadic behaviour. This is explained in his paper “Sporadic homogeneous structures” listed below.

So what is relational complexity? Given a group G acting on a set Ω, the *relational complexity* of this action is a particular positive integer which we denote $\mathrm{RC}(G, \Omega)$. This integer can be defined in a number of different ways, one of which involves the notion of a “homogeneous relational structure” – this is a combinatorial structure rather like a graph.

However one defines $\mathrm{RC}(G, \Omega)$, the problem of actually calculating this integer for a particular group is often rather tricky. For instance, it turns out that the smallest possible value for $\mathrm{RC}(G, \Omega)$ is 2... but the problem of working out which groups G satisfy $\mathrm{RC}(G, \Omega) = 2$ is still open. There has been recent progress on this problem however: the second reference listed below describes this progress in some detail.

There are many questions about relational complexity that warrant investigation: can we develop an algorithm for efficiently computing $\mathrm{RC}(G, \Omega)$? Can we say more about the situation when $\mathrm{RC}(G, \Omega) = 2$? Can we calculate which **simple** permutation groups G satisfy $\mathrm{RC}(G, \Omega) = 3$? Can we calculate the relational complexity of our favourite family of finite permutation groups? Any of these questions could form the basis of a PhD project in this area; all fit into the general programme of trying to use relational complexity to further our understanding of finite permutation groups.

Background reading/references

- G. Cherlin, Sporadic homogeneous structures, in *The Gelfand Mathematical Seminars*, Birkhauser, 2000, 15–48.
- N. Gill, M. Liebeck, P. Spiga, Cherlin’s conjecture on finite primitive binary permutation groups, 2021, preprint available at https://arxiv.org/abs/2106.05154. The first chapter is particularly relevant.