**School of Mathematics and Statistics**
**Faculty of Science, Technology, Engineering and Mathematics**

The Open University

# 2023 PhD Projects

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| **Project title** | Transcendental dynamics: Hausdorff dimension and itineraries |
| **Supervisors** | Prof Phil Rippon and Prof Gwyneth Stallard |
| **Discipline** | Pure mathematics |
| **Research area/keywords** | complex analysis, iteration, Hausdorff dimension |
| **Suitable for** | Full time applicants |

**Project background and description**

Complex dynamics concerns the iteration of analytic functions of the complex plane. For each function, the plane is split into two sets: the Fatou set (where the behaviour of the iterates is stable under local variation) and the Julia set (where the behaviour is chaotic). A good introduction to the main ideas of the theory for transcendental functions can be found in the article [1] by Bergweiler.

One of the most active areas of study in the subject has been the size of the Julia set (which is usually a fractal) and of sets of points with certain prescribed dynamical behaviour. For example, there has been much interest in the size of the set of points which escape to infinity under iteration, especially those which escape as fast as possible.
There are different ways of measuring the size of these fractals but, in transcendental dynamics, the most commonly used measure is the Hausdorff dimension. Most work in this area has concentrated on functions in a class known as the Eremenko-Lyubich class but, following the breakthrough paper [2], progress has begun to be made on more general classes of transcendental functions. The complex dynamics group at the OU was funded by the EPSRC to drive this forwards, leading to a number of papers including [4].
This PhD project will build on the work from the earlier EPSRC funded project, looking at sets of points with specified orbits defined in relation to a partition of the plane into a collection of annuli. The notion of an annular itinerary was introduced in [3] and has many applications.

**Background reading/references**


- W. Bergweiler, Iteration of meromorphic functions, *Bull. Amer. Math. Soc.*, 29 (1993), 151–188.
- W. Bergweiler, B. Karpińska, On the Hausdorff dimension of the Julia set of a regularly growing entire function, *Math. Proc. Cambridge Philos. Soc.*, 148 (2010), no. 3, 531–551. arXiv: 0909.3988
- P.J. Rippon and G.M. Stallard, Annular itineraries for entire functions, *Trans. Amer. Math. Soc.*, 367 (2015), no. 1, 377–399. arXiv: 1301.1328
- D. J. Sixsmith, Dimensions of slowly escaping sets and annular itineraries for exponential functions, *Ergodic Theory Dynam. Systems*, 36 (2016), no. 7, 2273–2292. arXiv: 1407.4638