Project title: Invariant measures for dynamical systems of algebraic origin

Principal supervisor: Reem Yassawi
Second supervisor: Ian Short

Discipline: Pure / Applied mathematics
Research area/keywords: Measurable preserving systems, Markov subgroups

Suitable for: Full-time or part-time applicants

Project background and description:
For a discrete dynamical system \((X, G)\), where \(G\) is a group acting on the space \(X\), a fundamental problem is to classify the measures \(\mu\) that are invariant under the action of \(G\). We say that \(\mu\) is trivial if it only gives positive mass to a periodic trajectory.

In this project, we seek to find nontrivial invariant measures of a class of dynamical systems called Markov subgroups, the most illustrious of which is the Ledrappier shift. For this system, which can also be defined using a cellular automaton, explicit non trivial invariant measures are given by the uniform Bernoulli measure, that is, the measure obtained using a sequence of identical fair coins. Recently, other measures have been identified for Markov subgroups generated by cellular automata. The difficulty is in showing that these measures are indeed nontrivial. They usually are supported on sets as in the figure below.

A point in the Markov subgroup generated by the local constraint \(1 + x + x^2 + y + xy + y^2 = 0\)

In this project we will extend these recent results to general Markov subgroups.

The project involves two components. One is writing and implementing algorithms to experimentally predict whether the measures are indeed nontrivial. The second component
involves proving that the obtained measures are nontrivial. This aspect of the project involves mastering some of the theory of Markov subgroups, and also that of automatic sequences.

Background reading/references


How to apply for this project

1. Read the Guide for applicants to check eligibility, especially entry and English language requirements.
2. Informal enquiries can be directed to the Director of Research.
3. Complete an application form, and send to the Director of Research by 6 March 2020.