Project title: Symmetries of weakly mixing substitution shifts

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Second supervisor: Uwe Grimm

Discipline: Pure / Applied mathematics

Research area/keywords: Measurable preserving systems, automorphism groups, joinings

Suitable for: Full time applicants

Project background and description

For a discrete dynamical system \((X, T)\), where \(T\) is an invertible continuous transformation acting on \(X\), we are interested in determining its group of symmetries, namely, the bijective continuous maps which commute with \(T\).

Clearly, any power of \(T\) must commute with \(T\), so at the very least, the symmetry group contains a copy of the integers \(\mathbb{Z}\). Some dynamical systems admit many other symmetries. For example, an irrational rotation of the unit circle commutes with any other irrational rotation. Circle rotations are isometries, and it is this property which allows for many symmetries.

On the other hand, some dynamical systems admit no other symmetries. When looking for such systems, we choose dynamical properties which are disjoint from the property of being an isometry, and one such property is that of a dynamical system being weakly mixing, which means that, on the average, two events become independent.

In this project, we seek to investigate the symmetry groups of weakly mixing dynamical systems generated by substitutions. We will focus on substitutions defined on a two-letter alphabet. To do this, we will investigate when such a dynamical system has minimal self-joinings, which is a kind of rigidity on how we can build products of measurable dynamical systems using \((X, T)\).

For this project, the student will first learn about basic mixing notions in ergodic theory, and substitution theory, and then apply this knowledge to characterise when a substitution on two letters is weakly mixing. They will then study two relevant articles, which concern one weakly mixing substitution, and generalise these articles to a family of substitutions.

Background reading/references


**How to apply for this project**

1. Read the **Guide for applicants** to check eligibility, especially entry and English language requirements.

2. Informal enquiries can be directed to the **Director of Research**.

3. Complete an **application form**, and send to the **Director of Research** by 6 March 2020.